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Mathematical modelling of moisture and heat transfer processes in soils for crop irrigation optimisation

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Abstract. The relevance of this study is conditioned by the need to develop scientifically sound methods to optimise the processes of moisture and heat transfer in soils, which will increase the efficiency of water resources use and improve crop yields. Under the conditions of global climatic changes and increasing deficit of water resources, effective management of crop irrigation becomes one of the key tasks of agronomy. The purpose of the present study was to create mathematical models describing the processes of moisture and heat transfer in soils to be applied to optimise irrigation regimes. The study highlighted the significance of integrating mathematical modelling into practical activities to achieve sustainable development of the agronomy sector. Convective diffusion processes in soils attract the attention of scientific researchers, as it is associated with the broad penetration of mathematical research methods in various fields of sciences. The great spread of surface irrigation methods necessitates the creation of mathematical models and techniques for their solution, revealing the principal regularities of both filtration and purely hydrodynamic processes. The study presented the key physical and mathematical regularities

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underlying the processes of moisture transfer and heat transfer in soil, as well as their interaction. The principal result of the study was a plant and its optimal development, which is of final economic interest, and the object of the study was soils, for which mechanical and mathematical models of the joint movement of moisture and heat with their different characteristics were developed. This study investigated the non-stationary convective diffusion equation. Based on the small perturbation method, the considered equation was represented as a linear equation and its solution was found in the automodel form and two classes of solutions were determined. The findings of this study may be useful for agronomists, engineers, and agricultural specialists seeking to implement modern water management technologies and improve the efficiency of agricultural production

Keywords: convective diffusion equation; hypergeometric Gauss equation; diffusion coefficient; Kummer's transformations; filtration; nonstationary equation

Introduction

The relevance of the subject is driven by the growing demands for efficient use of water resources in agriculture, especially in the context of climate change and increasing global population. Optimisation of irrigation is a key factor for increasing crop yields and sustainability, which requires a thorough understanding of soil processes. Mathematical modelling allows not only predicting the behaviour of moisture and heat in soils but also developing adaptive irrigation strategies that account for the specifics of various crops and climatic conditions. This contributes to more rational water use, cost reduction, and increased agricultural productivity, which is crucial for food security.

V. Díaz-González *et al.* (2022) performed numerical simulations of the proposed model to represent soil moisture content and plant growth dynamics and their consistency with the results of mathematical analysis. Furthermore, a concrete calibration was considered for field data obtained from a wheat experiment, and then these parameters were used to test the proposed model, resulting in an error in the estimation of soil moisture content of about 6%. D. Vishwakarma *et al.* (2023) argued that this simple model, which only requires soil, irrigation, and modelling parameters, is a valuable and practical tool for irrigation design. In this case, the net yield profitability increases markedly compared to the conventional method. It was also shown that the optimum irrigation scheduling solution strongly depends on the particular hydraulic properties of the soil at a given field site (Fontanet *et al.*, 2022).

The experimental findings of Z. Zhai *et al.* (2021) showed that the recommended irrigation rate is compared with the local experience of cultivation technology to obtain a decision accuracy of 81.7%. The irrigation and fertiliser management plan obtained from the intelligent decision-making system provides higher plant height during the growth period than that of locally grown crops. I. Lakhiar *et al.* (2024) concluded that the future of precision irrigation and water conservation systems looks bright due to the need for efficient irrigation water management systems, technological advancements, and increased environmental awareness. As water scarcity is exacerbated by climate change and population growth, precision irrigation and water conservation systems are poised to play a crucial role in optimising and modernising water use, improving water use efficiency and reducing adverse environmental impacts, thereby ensuring sustainable agricultural development.

S. Jaiswal & M. Ballal (2020) presented an automated irrigation controller based on real-time fuzzy inference implemented in LabVIEW, which utilises data from sensor network and transmitted using GSM/GPRS module. An efficient and sustainable irrigation mechanism results in reduced water losses and increased crop yields. O. Novikov *et al.* (2021) proved that the development of irrigation requires a systematic approach with mandatory scientific support on the terms of public-private partnership and can increase revenues to the state budget.

Tremendous attention in reclamation and intensification of agriculture is paid to improving the efficiency of irrigated land use, creation of progressive irrigation methods, improvement of irrigation techniques and technology, significant saving of water reserves, preservation of existing agricultural lands. The purpose of the present study was to develop and apply mathematical models for the analysis and optimisation of moisture and heat transport processes in soils, which will improve water resources management in agriculture.

Materials and Methods

Mathematical modelling was the principal method employed in the present study, which helped to create abstract models describing the processes of moisture and heat transfer in soil. Comparative analysis was used to compare various irrigation methods and their effects on crop growth, which can help in selecting the best approach. Using the systemic approach, processes were considered in the context of the entire agro-ecosystem, including interactions between soil, plants, and the atmosphere. These methods enabled a deeper understanding of soil processes and the optimisation of irrigation systems to improve agricultural efficiency.

Groundwater always contains a certain amount of dissolved salts. A certain amount of salts are in the soil, in the solid phase, can be sorbed on soil particles, be dispersed within soil pores, and appear as a result of irrigation (Hakami *et al.*, 2024). When groundwater rises, moisture approaches the soil surface at a fairly close distance, intense evaporation takes place, and salts are carried to the upper parts of the soil and to the soil surface. At the same time, soil fertility decreases and after a certain period the utilised land may become infertile (Li *et al.*, 2025). Therefore, methods of investigating the water-salt regime of soils and subsoils are essential for land reclamation (Bajpai & Kaushal, 2020). Both small and large portions of water, food, heat, etc., are equally bad for the plant.

Thus, the movement of salts in soils is driven by their migration with solutes. This process of salt transfer in soils, in the absence of chemical reaction and sorption, is described by the following equation (1):

$$m_0\frac{\partial C}{\partial t}=\frac{\partial}{\partial x}\left(D(C)\frac{\partial C}{\partial x}\right)+ +\frac{\partial}{\partial y}\left(D(C)\cdot\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(D(C)\frac{\partial C}{\partial z}\right)-\operatorname{div}(\mathrm{vC}), \qquad (1)$$

where C (x, y, z, t) is the salt concentration in the liquid, v is the filtration speed, m_0 is the active porosity of soil, which characterises the active volume of the pore space, D (C) is the diffusion coefficient, which in the general case is a function of the salt concentration in the liquid (Derbie *et al.*, 2024). It was previously proposed that the soil was assumed to be homogeneous, isotropic, and therefore the diffusion coefficient and filtration rate components were considered constant in the first approximation (Henri & Diamantopoulos, 2023). In this case, equation (1) had the following form:

$$m_0\frac{\partial C}{\partial t}=D_0\left[\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right]- -U_0\frac{\partial C}{\partial x}-V_0\frac{\partial C}{\partial y}-W_0\frac{\partial C}{\partial z}\quad , \qquad (2)$$

while the following initial-boundary conditions were set:

a. At the initial moment $t = t_0$, C $(x, y, x, t_0) = P_0$ (x, y, z) (3)

b. At the boundary C $(x, y, x, t_0) = Q_0$ (x_0, y_0, z_0, t). (4)

Next, a new function was introduced:

$$C\,(x, y, x, t_0) = exp\xi_0\cdot Q\,(x, y, x, t),\ \xi_0 = = a_0\,x + b_0\,y + c_0\,z + d_0\,t, \qquad (5)$$

and a linear equation of the following form was obtained:

$$Q_\tau = Q_{xx} + Q_{yy} + Q_{zz}, \qquad (6)$$

which is an equation for further studies, therewith:

$$a_0=\frac{U_0}{2\mathrm{D}_0},\ \mathrm{b}_0=\frac{V_0}{2\mathrm{D}_0},\ \mathrm{C}_0=\frac{W_0}{2\mathrm{D}_{,0}},$$

$$\mathrm{d}_0=-\frac{U_0^2+V_0^2+W_0^2}{4\mathrm{m}_0 D_0},\ \tau=\frac{D_0}{m_0}\mathrm{t}.$$

The solution of equation (6), invariant relative to the uniparametric similarity transformation group:

$$x\Rightarrow\gamma x,\ y'\Rightarrow\gamma y,\ z'\Rightarrow\gamma z,\ \tau'\Rightarrow\gamma\tau,\ Q^1\Rightarrow\gamma Q,$$

which must have the following form:

$$Q(\mathrm{x},\mathrm{y},\mathrm{z},\tau_0)=\tau^n\cdot P(\eta_1),\ \eta_1=\frac{(x^2+y^2+z^2)}{\tau}. \qquad (7)$$

By finding all necessary specific derivatives and substituting them into the equation (6) under study, the following ordinary differential equation can be obtained:

$$P''(\eta_1)+\left[\frac{1}{2}+\frac{1}{4}\eta_1\right]P'(\eta_1)-\frac{n}{4}P(\eta_1)=0. \qquad (8)$$

Comparing it with an equation of the form

$$xy'' + (ax + b)\,y' + (cx + d)y = 0, \qquad (9)$$

it is possible to write one of the solutions as follows:

$$y=x^{-b/2}\cdot e^{-ax/2}\cdot F\left(\frac{2d-ab}{2\sqrt{a^2-4c}}\right), \quad \frac{1}{2}(b-1);\ x\sqrt{a^2-4c} \qquad (10)$$

i.e., one of the specific solutions of equation (8) takes the following form:

$$P(\eta_1)=\eta_1^{-1.4}\cdot e^{-\eta_1/8}\cdot F\left(-\frac{4\mathrm{n}+1}{4},-\frac{1}{4};\frac{1}{4}\eta_1\right), \qquad (11)$$

where $a=\frac{1}{4}$, $b=\frac{1}{2}$, $c=0$, $d=-\frac{\kappa}{4}$.

The last solution can be written as a polynomial at $\kappa = (4n - 1) / 4$. The solution of the studied equation (6), in general form, is written as follows $\eta_1=\frac{(x^2+y^2+z^2)}{\frac{D_0}{m_0}t}$.

$$C\,(x, y, z, t)=exp\,\eta_1\cdot\left(\frac{D_0}{m_0}t\right)^n\cdot \cdot\left[\eta_1{}^{-1/4}\right]\cdot exp\,\eta_1\cdot F\left(-\frac{4n+1}{4},\frac{1}{4},\frac{1}{4}\eta_1\right). \qquad (12)$$

If a new variable $\eta = -\eta 1/4$ is introduced, then defining the first and second derivatives P_1', P_1'' and substituting into (8), the following expression is obtained:

$$\eta\ \mathrm{P}''(\eta)+\left[\frac{1}{2}-\eta\right]\cdot P_1'(\eta)+\mathrm{nP}_1(\eta)=0, \qquad (13)$$

which is a degenerate hypergeometric Gauss equation and has two linearly independent solutions:

$$P_1(\eta)=C_1F_1\left(-n,\frac{1}{2};\eta\right)+C_2\eta^{1/2}F_2\left(-\eta+\frac{1}{2},\ \frac{3}{2};\eta\right). \qquad (14)$$

It is known that both linearly independent solutions simultaneously cannot be written as an algebraic polynomial. The first solution has the form of a polynomial when $n = 1, 2, 3....K$, and the second when $n=\frac{3/2,5/2,\ldots\,(2\kappa+1)}{2}$.

Results and Discussion

Some exact solutions can be written for the first solution of equation (6) considering the planned solution (7)

$$Q\,(x,y,z,\tau) = C_1\,(2\tau + \eta^2),$$
$$at\ n = 1,$$

$$Q\,(x,y,z,\tau) = C_1\,(12\tau^2 + 12\tau\eta + \eta^4),$$
$$at\ n = 2,$$

$$Q\,(x,y,z,\tau) = C_1\,(120\tau^3 + 180\tau^2\,\eta + 30\,\tau\eta^4 + \eta^6),$$
$$at\ \text{n} = 3,$$

$$Q\,(x,y,z,\tau) =$$
$$= C_1\,(a_0\tau^k + a_1\tau^{k-1}\eta + \cdots a_{n-1}\tau\eta^{2n-2} + a_n\,\eta^{2n}),\ at\ n = k. \quad (15)$$

For the second linearly independent solution of equation (6), there will be other expressions $Q\,(x,y,z,\tau) = C_2\,\eta\,(6\tau + \eta^2)$, $at\ n = 3/2$,

$$Q(x,y,z,\tau) = C_2\eta\,(60\tau^2 + 20\tau\eta^2 + \eta^4),\ at\,n = {}^5/_2,$$

$$Q\ (x,y,z,\tau) =$$
$$= C_2\eta\,(80\tau^3 + 420\tau^2\eta^2 + 42\tau\eta^4 + \eta^6),\ at\,n = 7/2$$

$$Q\ (x,y,z,\tau) = C_2\dot{\eta}\ \times$$
$$\times\big(b_0\tau^{k\text{-}1/2} + b_1\tau^{k\text{-}3/2}\eta^2 + \cdots b_{n\text{-}1}\tau\eta^{2k\text{-}3/2} + b_n\eta^{2k+1/2}\big),$$
$$at\,n = n + 1/2 \quad (16)$$

Thus, the authors of this study developed two classes of specific automodel solutions for equation (6), which have many different solutions at varying n values. Using Kummer's transformations, one can find other classes of specific solutions (Kamke, 1977).

A different solution of equation (6) was also considered, based on the group properties of differential equations, where the invariance of the equation under study relative to the transformation group of independent and dependent variables was observed (Singh *et al.*, 2020). Based on the above, the solution of equation (6) can be found in the following form:

$$Q\ (x,y,z,\tau) =$$
$$= (x+y+z)^2 \cdot f(\xi_1),\ \text{where}\ \xi_1 = \frac{\tau}{(x+y+z)^2} \quad (17)$$

By finding all the necessary specific derivatives and substituting them into the equation, an ordinary differential equation of the second order can be obtained

$$\xi_1^2 f_0{}''(\xi_1) -$$
$$-\left[\frac{1}{12} + \left(n - \frac{3}{2}\right)\xi_1\right] f'(\xi_1) + \frac{n(n-1)}{4} f(\xi_1) = 0. \quad (18)$$

With the introduction of the new function

$$f'(\xi_1) = e^{\xi}\cdot\xi^{\nu} f_0(\xi),\ \text{where}\ \xi = \xi_1^{-1}, \quad (19)$$

the following equation will be obtained:

$$\xi^2 f_0{}''(\xi) + \left[\frac{25}{12}\xi^2 + \left(2\nu + \frac{1}{2} + n\right)\xi\right] f_0'(\xi)$$
$$+\left[\frac{13}{12}\xi^2 + \left(n + \frac{1}{2} + \frac{25}{12}\nu\right)\xi\right]\cdot f_0(\xi) +$$
$$+\left[\frac{n(n-1)}{4} + \left(n - \frac{1}{2}\right)\nu + \nu^2\right] f_0(\xi) = 0. \quad (20)$$

Assuming that the square bracket of the last term of the resulting equation is zero, this is possible at $v_1 = -n/2$ and $v_2 = (1 - n)/2$" and so for each value of v the following two equations will be called degenerate hypergeometric Gauss equations:

$$\Xi f_0{}'' + \left[\frac{1}{2} + \frac{25}{12}\xi\right] f_0' +$$
$$+\left[\frac{13}{12}\xi - \frac{n-12}{24}\right] f_0 = 0,\ \text{at}\ \nu_1 = -\frac{n}{2} \quad (21)$$

$$\xi f_0{}'' + \left[\frac{3}{2} + \frac{25}{12}\xi\right] f_0' +$$
$$+\left[\frac{13}{12}\xi - \frac{n-37}{24}\right] f_0 = 0,\ \text{at}\ \nu_1 = -\frac{n\text{-}1}{2}. \quad (22)$$

In the vicinity of the singular point $\xi = 0$, both linearly independent solutions of equation (21) will be written as follows:

$$f_0(\xi) = exp\left(-\frac{13}{12}\xi\right)\cdot$$
$$\cdot\left[C_1 F\left(\frac{n+1}{2}, \frac{1}{2}; \frac{\xi}{12}\right) + C_2\xi^{1/2} F\left(\frac{n+2}{2}, \frac{3}{2}; \frac{\xi}{12}\right)\right] \quad ,(23)$$

where C_1, C_2 are integration constants.

The general solution of equation (6), considering the solution of (17, 19), will take the following form:

$$Q\ (x,y,z,\tau) = \tau^{n/2}\cdot exp(-\xi/12)\cdot$$
$$\cdot\left[C\ {}_1F\left(\frac{n+1}{2}, \frac{1}{2}; \frac{\xi}{12}\right) + C_2\xi^{1/2}\cdot F\left(\frac{n+2}{2}, \frac{3}{2}; \frac{\xi}{12}\right)\right] (24)$$

Thus, the required function for equation (21) will finally be written as follows:

$$N(x,y,z,t) = exp\,(a_0x + b_0y + c_0z - d_0t)\cdot\left(\frac{D_0}{m_0}t\right)^{n/2}$$
$$\cdot\ exp\left(\frac{m_0(x+y+z)^2}{D_0+12t}\right)\cdot$$
$$\cdot\left\{\begin{array}{c} C_1F\left[\frac{n+1}{2}, \frac{1}{2}; \frac{m_0}{12D_0}\cdot\left(\frac{x+y+z}{t}\right)^2\right] + \\ + C_2\left(\frac{m_0}{12D_0}\frac{(x+y+z)^2}{t}\right)^{1/2} F\left[\frac{n+2}{2}, \frac{3}{2}; \frac{m_0}{12D_0}\frac{(x+y+z)^2}{t}\right]\end{array}\right\} \quad (25)$$

The general solution for equation (22) will take the following form:

$$f_0(\xi) = exp\left(-\frac{13}{12}\xi\right)\cdot$$
$$\cdot\left[C_1 F\left(\frac{n+2}{2}, \frac{3}{2}; \frac{\xi}{12}\right) + C_2\xi^{\text{-}1/2} F\left(\frac{n+1}{2}, \frac{1}{2}; \frac{\xi}{12}\right)\right], \quad (26)$$

then for equation (6) itself, the solution will be written as follows:

$$Q(x,y,z,\tau)=\left(\frac{m_0}{D_0}\tau\right)^{\frac{n-1}{2}}(x,y,z,\tau)exp\left(-\frac{\xi}{12}\right)\cdot$$
$$\cdot\left[C_1F\left(\frac{n+2}{2},\frac{3}{2};\frac{\xi}{12}\right)+C_2\xi^{-1/2}F\left(\frac{n+1}{2},\frac{1}{2};\frac{\xi}{12}\right)\right] \quad (27)$$

and considering expression (5), ultimately there is

$$\mathrm{C}(x,y,z,t)=exp\,(a_0x+b_0y+c_0z-d_0t)\cdot$$
$$\cdot\,t^{\frac{n-1}{2}}(x+y+z)\cdot exp\left(-\frac{m_0}{12D_0}\frac{(x+y+z)^2}{t}\right)\cdot$$
$$\cdot\left[C_1F\left(\frac{n+2}{2},\frac{3}{2};\frac{m_0}{12D_0}\frac{(x+y+z)^2}{t}\right)+C_2\left(\frac{m_0}{12D_0}\frac{(x+y+z)^2}{t}\right)^{-1/2}\cdot\right.$$
$$\left.\cdot\,F\left(\frac{n+1}{2},\frac{1}{2};\frac{m_0}{12D_0}\frac{(x+y+z)^2}{t}\right)\right] \quad (28)$$

Next, the analysis will be made when each specific solution of the degenerate hypergeometric Gaussian function can be represented in the form of an algebraic expression. The first specific solution of expression (23) is represented in the form of a polynomial when $n=-2k+1$, and the second at $n=-2k-2$.

Hence, it can be observed that at the same values of the automodelling index, the specific solutions cannot be simultaneously written in algebraic polynomials. Some exact solutions of equation (6) will be defined, for the first specific solution:

$$C(x,y,z,t)=exp\,(a_0x+b_0y+c_0z-d_0t)\cdot$$
$$\cdot\left(\frac{D_0}{m_0}t\right)^{-1/2}\cdot exp\left(-\frac{m_0(x+y+z)^2}{12D_0t}\right) \quad (29)$$

$$\mathrm{C}(x,y,z,t)=$$
$$=exp\,(a_0x+b_0y+\mathrm{c}_0z-d_0t)\cdot\left(\frac{D_0}{m_0}t\right)^{-3/2}\cdot$$
$$\cdot\;exp\left(-\frac{m_0(x+y+z)^2}{12D_0t}\right)\cdot\left[1-\frac{m_0(x''+y+z)^2}{6D_0\cdot t}\right] \quad (30)$$

Next, two solutions at $n=-2$ and $n=-4$ will be written out for the second specific solution:

$$C(x,y,z,t)=C_2\left(\frac{D_0}{m_0}t\right)^{3/2}\cdot$$
$$\cdot\,(x+y+z)\;exp\;\cdot$$
$$\cdot\left[(a_0x+b_0y+c_0z+d_0t)-\frac{m_0(x+y+z)^2}{12D_0t}\right] \quad (31)$$

$$C(x,y,z,t)=C_2\left(\frac{D_0}{m_0}t\right)^{5/2}\cdot(x+y+z)\;exp\cdot$$
$$\cdot\left[(a_0x+b_0y+c_0z-d_0t)-\frac{m_0(x+y+z)^2}{12D_0t}\right]\cdot$$
$$\cdot\left[1-\frac{m_0(x+y+z)^2}{18D_0t}\right] \quad (32)$$

The authors of the present study defined a class of exact solutions of equation (2,6), and the analysis of the obtained solutions convinced specialists that the number of summands obtained by polynomials increases with decreasing of the automodelling index n. The integration constants C_1, C_2 are determined from the explicit setting of the initial boundary conditions (3, 4). The principal result of the study was the optimum development of various crops in the field, which is of final economic interest, and the object of the study was soil, for which mechanical and mathematical models of the joint movement of moisture, heat, and various concentrations of salts with different characteristics were developed.

Based on the principles of geochemical hydrodynamics and the results of its application in the field of irrigation and land drainage, the study investigated and outlined the methodology for calculating the salt regime of soils (Rachinskaya *et al.*, 1963). According to this, the equation of convective diffusion and mass transfer during water filtration in soils was analysed analytically (Sagyndykova & Tuganbaev, 2014), and methods of calculation of these processes were proposed. In irrigated areas, a significant problem is the prevention of salinisation of fertile lands that were abandoned due to the rise of saline groundwater and other types of salinisation (Phogat, 2012). According to W. Guo *et al.* (2001), salinisation of naturally saline lands, which after desalinisation become suitable for farming, continues to be relevant.

Therefore, measures preventing soil salinisation, forecasting, and correct calculations of the consequences of land irrigation are of paramount significance. M. Hagage *et al.* (2024) pointed out that the application of reclamation measures substantially affects the natural hydrochemical regime of the upper part of the soil, which can cause severe environmental consequences. Therefore, the development of sound methods for predicting hydrochemical processes in soils, aeration zone soils, and groundwater is critical (Zhai *et al.*, 2021). The prediction should explain why the same intervention, in some conditions gives a strong effect, while in other conditions turns out to be suboptimal and sometimes harmful. According to H. Li & Z. Luo (2011), the correct forecast is based on the methods of geochemical hydrodynamics, which combines the principles of filtration theory, diffusion, chemical kinetics, and modern mathematics.

A. Kostyakov (1960) attempted to uncover the mechanism of salt movement in soil and to show quantitative interaction of factors. The researcher used the equation to calculate the increase of groundwater salinity where U is the rate of groundwater evaporation, C is the salt concentration, v is the diffusion coefficient, x is the coordinate directed downwards, t is time, $n=0$; 0.5;1. This equation is derived from the equality in the equilibrium case of two salt fluxes (convective and diffusive) in an arbitrary soil section parallel to the free surface. However, the theoretical dependence obtained from this equation cannot claim to be general at least because it is obtained based on hypotheses with certain assumptions. Assumptions that it is possible to find a single dependence for the entire zone of salt transfer, regardless of the soil structure and the nature of the moisture regime in it, do not correspond to reality.

L. Rex (1968) offered analytical solutions of the system of two equations for the case of steady and unsteady groundwater regime, which allow making forecasts of soil salt regime under irrigation. However, in practice, both progressive rise of groundwater table in case of irrigation and its marked decrease in case of evaporation take place. The process is non-stationary. Furthermore, the models of the above researchers, when considering salt and moisture migration in soil under natural conditions and under irrigation, did not account for horizontal movement of groundwater flow and did not determine the relationship between existing water-salt regimes of soils and groundwater flow. At the same time, groundwater velocity and salinity substantially influence the distribution of salt concentrations in soil (Xun *et al.*, 2022). Notably, consideration of solely the vertical movement of soil moisture leads to the necessity of setting a concentration value at some depth, which is factually an uncertain variable.

Conclusions

The mathematical model of uniform soil hydration was theoretically substantiated, spatial mechanical and mathematical model of heat and moisture transfer and salt migration in different soils was developed and their approximate analytical solutions under initial boundary conditions with different physical and mechanical properties of soil were obtained. Since in the Kyrgyz Republic surface irrigation is applicable for any agricultural crop and is the most widespread, it comes with both advantages and disadvantages. In this regard, quantitative hydrodynamic studies on the selection of the principal elements of irrigation technology, theoretical study of the processes of heat, moisture, and salt transfer in the soil, to create the best conditions for plant development are significant.

Irrigation is an essential factor for increasing crop yields, especially under varying climate conditions. Modelling of moisture and heat transfer processes helps to better understand the interaction between soil, water, and plants. The study described the mathematical models used to simulate moisture and heat transport processes. These models account for soil physicochemical properties, climatic conditions, and plant needs. The models allow predicting the distribution of moisture and temperature in the soil, which contributes to more efficient irrigation scheduling. The findings showed how different irrigation regimes affect soil conditions and plant growth.

Based on the data obtained, it is possible to develop recommendations for optimising irrigation regimes, which reduces water losses and improves conditions for crop growth. Agronomists and farmers can use the findings of this study to make more informed decisions in irrigation management, which can lead to improved agricultural sustainability. These findings highlight the significance of mathematical modelling in agronomy and its contribution to sustainable agriculture. Therewith, there is a need for further research to improve models and account for other factors such as climate change and soil characteristics.

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Conflict of Interest

The authors of this study declare no conflict of interest.

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Айыл чарба өсүмдүктөрүн сугарууну оптималдаштыруу үчүн жер кыртышындагы ным жана жылуулук өткөрүү процесстерин математикалык моделдөө

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Аннотация. Бул иштин актуалдуулугу жер кыртышындагы ным жана жылуулук берүү процесстерин оптималдаштыруунун илимий жактан негизделген ыкмаларын иштеп чыгуу зарылдыгы менен шартталган, бул суу ресурстарын пайдалануунун натыйжалуулугун жогорулатууга жана түшүмдүүлүктү жакшыртууга мүмкүндүк берет. Глобалдык климаттык өзгөрүүлөрдүн жана суу ресурстарынын өсүп бараткан тартыштыгынын шартында айыл чарба өсүмдүктөрүн сугарууну натыйжалуу башкаруу – агрономиянын негизги милдеттеринин бири болуп калууда. Бул иштин максаты сугат режимдерин оптималдаштыруу үчүн жер кыртышындагы нымдуулук жана жылуулук берүү процесстерин сүрөттөгөн математикалык моделдерди түзүү болгон. Макалада агросектордун туруктуу өнүгүүсүнө жетишүү үчүн математикалык моделдөөнү практикалык иш-аракетке интеграциялоонун маанилүүлүгү баса белгиленген. Жер кыртышындагы конвективдүү диффузия процесстери илимий изилдөөчүлөрдүн көңүлүн бурат, анткени бул илимдин ар кандай тармактарына математикалык изилдөө ыкмаларынын кеңири жайылышына байланыштуу. Жер үстүндөгү сугат режимдеринин кеңири жайылышы чыпкалоо жана гидродинамикалык процесстердин негизги үлгүлөрүн ачып бере турган математикалык моделдерди, аларды чечүү ыкмаларын түзүү зарылдыгын шарттайт. Макалада топурактагы ным жана жылуулук берүү процесстеринин, ошондой эле алардын өз ара аракеттенүүсүнүн негизги физикалык жана математикалык мыйзам ченемдүүлүктөрү келтирилген. Изилдөөнүн негизги натыйжасы өсүмдүк жана анын оптималдуу өнүгүшү болгон, ал акыркы экономикалык кызыгууну жаратат, ал эми изилдөө объектиси ар кандай мүнөздөмөлөрү менен нымдуулуктун жана жылуулуктун биргелешкен кыймылынын механикалык-математикалык моделдери иштелип чыккан жер кыртышы болгон. Бул жерде конвективдүү диффузиянын стационардык эмес теңдемеси изилденген. Чакан толкундоолор ыкмасынын негизинде, каралып жаткан теңдеме сызыктуу деп көрсөтүлөт жана анын чыгарылышы автомодельдик түрдө табылып, чыгарылыштардын эки классы аныкталат. Изилдөөнүн натыйжалары агрономдор, инженерлер жана айыл чарба адистери үчүн пайдалуу болушу мүмкүн, алар заманбап суу ресурстарын башкаруу технологияларын киргизүүгө жана агроөнөр жай өндүрүшүнүн натыйжалуулугун жогорулатууга умтулушат

Негизги сөздөр: конвективдүү диффузия теңдемеси; Гаусстун гипергеометриялык теңдемеси; диффузия коэффициенти; Куммердин өзгөртүүсү; чыпкалоо; стационардык эмес теңдеме

Математическое моделирование процессов влаго- и теплопереноса в почвогрунтах для оптимизации орошения сельскохозяйственных культур

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Аннотация. Актуальность данной работы обусловлена необходимостью разработки научно обоснованных методов оптимизации процессов влаго- и теплопереноса в почвогрунтах, что позволит повысить эффективность использования водных ресурсов и улучшить урожайность. В условиях глобальных климатических изменений и нарастающего дефицита водных ресурсов эффективное управление орошением сельскохозяйственных культур становится одной из ключевых задач агрономии. Целью данной работы было создание математических моделей, описывающих процессы влаго- и теплопереноса в почвах, с целью их применения для оптимизации режимов орошения. В статье подчеркнута важность интеграции математического моделирования в практическую деятельность для достижения устойчивого развития агросектора. Процессы конвективной диффузии в почвогрунтах привлекают внимание научных исследователей, так как это связано с широким проникновением математических методов исследования в различные области наук. Большое распространение поверхностных способов орошения диктует необходимость создания математических моделей и методик их решения, позволяющих выявить основные закономерности как фильтрационных, так и чисто гидродинамических процессов. В работе представлены основные физические и математические закономерности, лежащие в основе процессов влагопереноса и теплопередачи в почве, а также их взаимодействия. Главным результатом исследования было растение и его оптимальное развитие, которое представляет итоговый экономический интерес, а объектом исследования были почвогрунты, для которых разработаны механико-математические модели совместного передвижения влаги и тепла с различными их характеристиками. В работе было исследовано нестационарное уравнение конвективной диффузии. На основании метода малых возмущений, рассматриваемое уравнение представлено как линейное, и его решение найдено в автомодельном виде и определены два класса решений. Результаты исследования могут быть полезны для агрономов, инженеров и специалистов в области сельского хозяйства, стремящихся к внедрению современных технологий управления водными ресурсами и повышению эффективности агропроизводства

Ключевые слова: уравнение конвективной диффузии; гипергеометрическое уравнение Гаусса; коэффициент диффузии; преобразования Куммера; фильтрация; нестационарное уравнение