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INCREASING OF HOLOGRAPHIC INTERFEROMETER SENSITIVITY BY DIGITAL METHODS

УЛУЧШЕНИЕ ЧУВСТВИТЕЛЬНОСТИ ГОЛОГРАФИЧЕСКОГО ИНТЕРФЕРОМЕТРА ЦИФРОВЫМИ МЕТОДАМИ

Ички параметрлери кескин өзгөрүүсү менен мүнөздөлүүчү, татаал фазалык чөйрөнү изилдөөгө багытталган, интерферометрдин сезгичтигин жөнгө салуучу ыкманы иштеп чыгуу өтө актуалдуу проблема болуп эсептелинет.

Изилдөөнүн максаты, макалада келтирилген натыйжалар, оптикалык маалыматты жөндөөдөгү санариптик ыкмаларды колдонуунун жардамы менен голографиялык интерферометрдин чыгуу параметринин жөндөмдүүлүгүн күчөтүү мүмкүнчүлүгүн анализдөө болуп саналат.

Изилдөөнүн натыйжасы, объектик жана таяныч толкундар, интерференциялык сүрөттөлүштүн капталган ПЗС-матрицасы жөнүндөгү санариптик маалыматтын бар болушу, ушул берилмелердин негизиндеги ар кандай математикалык операциялар жүргүзүлөт жана андан кийин аларды кайра экранга чыгарууга болот, же элестетүү мүмкүнчүлүгүнө ээ болот.

Операцияны объектен чыгуу тегиздигине чейинки бөлүгүндөгүдөй эле ПЗС матрица такталган интенсивдүүлүктүн системасында да жүргүзүүгө болот.

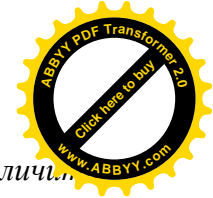
Голографиялык системанын сезгичтигин $2m$ эсеге жогорулатууга мүмкүн болуучу сандык операциялардын удаалаштыгы сунушталат, мында $m=1,2,3...$

Андан кийин көрсөтүлгөн схема боюнча сезгичтикти күчөтүү мүмкүн болгон спекл – чуулдоо гана чектелгендиги көрсөтүлгөн жана алар интерференциялык сүрөттөлүштүн тилкелеринин кескин айырмачылыгынын начарлашына дуушар кылат.

Өзөк сөздөр: голографиялык интерферометрия, голографиялык системанын сезгичтиги, ПЗС-матрица, цифралык усулдар, Френелдин кайра түзүүсү, Фурьенин кайра түзүүсү, объектулик толкун, тирөөч толкун.

Задача разработки эффективных методов повышения чувствительности голографических интерферометров необычайно важна при исследованиях сложных фазовых сред, характеризующихся сильно изменяющейся неоднородностью среды.

Целью данного исследования, результаты которого представлены в статье, является разработка метода повышения чувствительности голографических интерферометров посредством цифровой обработки выходных данных методами цифровой обработки оптической информации. Результаты исследований показали, что наличие цифровой информации об объектной и опорной волнах и об интерференционной картине, регистрируемой ПЗС-матрицей, позволяет выполнять над этими данными различные математические операции, после чего их можно снова отобразить на экране, т. е. визуализировать. Эти операции могут осуществляться как на участках от объекта до выходной плоскости, так и на выходе системы, непосредственно с распределением интенсивности, которое фиксируется ПЗС-матрицей.



Предложена последовательность численных операций, позволяющая увеличить чувствительность голографической системы в 2^m , где $m = 0, 1, 2, 3, \dots$. Показано также, что увеличение чувствительности по указанной схеме ограничивается только возможными спекл-шумами, которые могут ухудшать контрастность полос интерференционной картины.

Предлагаемый способ повышения чувствительности интерферометра, в отличие от известных, не основан на аппаратных изменениях в интерферометрических системах, что связано с большими затратами, а цифровыми методами улучшает характеристики выходных данных низкочувствительных интерферометров.

Результаты работы могут быть использованы при исследованиях сложных фазовых сред.

Ключевые слова: *голографическая интерферометрия, чувствительность голографической системы, ПЗС-матрица, цифровые методы, преобразование Френеля, преобразование Фурье, объектная волна, опорная волна.*

The development of methods for increasing the sensitivity of interferometers intended for studying complex phase media, which are characterized by abrupt changes in internal inhomogeneities, is a very urgent task.

The aim of the study, the results of which are presented in the article, is a theoretical analysis of the possibility of improving the sensitivity of the output data of a holographic interferometer using digital methods of processing optical information. The research results showed that the presence of digital information about the object and reference waves and about the interference pattern recorded by the CCD matrix allows performing various mathematical operations on these data, after which they can again be displayed on the screen, i.e., visualized. These operations can be carried out both in the sections from the object to the output plane, and at the output of the system, directly with the intensity distribution, which is fixed by the CCD matrix.

A sequence of numerical operations is proposed, which makes it possible to increase the sensitivity of the holographic system by a factor of 2^m , where $m = 0, 1, 2, 3, \dots$. It is also shown that the increase in sensitivity according to the indicated scheme is limited only by possible speckle noise, which can worsen the contrast of the fringes of the interference pattern.

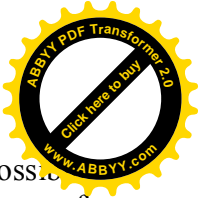
The proposed method for increasing the sensitivity of the interferometer, in contrast to the known ones, does not rely on hardware changes in interferometric systems, which is associated with high costs, but digitally improves the characteristics of the output data of low-sensitivity interferometers.

The results of the work can be used in studies of complex phase media.

Key words: *holographic interferometry, holographic system sensitivity, CCD matrix, digital methods, Fresnel transform, Fourier transform, object wave, reference wave.*

Introduction

Interferometric methods are well suited for measuring the optical densities of dynamic phase media like plasma or complex aerodynamic flows [1]. However, taking into account such a feature of such media, as, in most cases, the presence of only small gradients of the refractive index, it can be concluded that the sensitivity of classical interferometers is insufficient for recording the density distribution in such phase media [2]. In principle, the sensitivity of classical interferometry methods can be increased. It should be borne in mind that in classical interferometry, any change in the wavefront is determined by an optical method, which makes it possible to use precisely optical methods for increasing the sensitivity. First of all, this is an opportunity to increase the number of wave passes in the interferometer. The addition of various interference orders can be used, as is done in a multichannel holographic interferometer [3]. You can use the methods of multiwave interferometry, various nonlinear effects, etc. [4]. Moreover, all these methods can be used both to increase the sensitivity of classical interferometry methods,



and to reduce them [4]. However, the development of information technology makes it possible to achieve similar results through various transformations of the recorded and digitized image of the interference pattern. The transition to digital methods for processing interferograms is especially effective in holographic interferometry, since digital holographic methods are very well developed, and the methods of holographic interferometry themselves turn out to be much more sensitive. That is, it is possible to achieve the highest efficiency of the sensitivity of interference methods for measuring the parameters of phase media using, first of all, the methods of digital holographic interferometry.

The purpose of the study considered in the article is to theoretically substantiate the possibility of improving the sensitivity of digital holographic interferometry methods, which will allow one to determine very small fluctuations in the inhomogeneities of the phase media under study.

Theoretical substantiation of the possibility of increasing the sensitivity of the method of two exposures

Consider the classical scheme of recording a quasi-Fourier hologram. In this scheme, an off-axis point reference and an object, which in general can be either opaque or transparent, are placed on the same plane, which can be called the input plane of the system. In the case of digital holography, the plane of the CCD (charge coupled device) sensors of the matrix is usually taken as the output plane. Let us denote the distance between the input plane of the system and the plane of the CCD matrix by l . The coordinate system in the input plane will be denoted by (x_0, y_0) , in the output plane (x, y) . Mathematically, the input plane can be represented as [5, 6]

$$v(x_0, y_0, z_0) = \delta(x_0 - X, y_0 - Y) + u(x_0, y_0). \quad (1)$$

Here (X, Y) are the coordinates of the position of the point source in the input plane, $u(x_0, y_0) = a \exp[i\theta(x_0, y_0)]$ is the wave coming from the object, $\delta(x_0 - X, y_0 - Y)$ is the delta function describing the point source. The distance between the input and output planes is such that the plane in which the CCD is located marks the Fresnel region. That is, the field of a light wave in the plane of the CCD matrix can be found using the Fresnel approximation.

$$v_z(x, y, z) = \frac{\exp(ikz)}{ikz} \iint_{\infty} v(x_0, y_0, z_0) \exp\left\{\frac{i\pi}{\lambda z} [(x - x_0)^2 + (y - y_0)^2]\right\} dx_0 dy_0. \quad (2)$$

Equation (2) represents the diffraction integral in the form of the Fresnel transform, which is obtained as a paraxial approximation of the general diffraction integral [7].

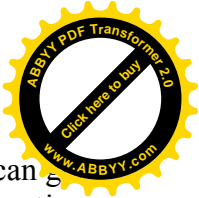
Here $k = \frac{2\pi}{\lambda}$ is the wave number, λ is the length of the light wave, z is the coordinate on the axis along which the light wave propagates.

The Fresnel transform can be reduced to the Fourier transform [8]

$$\begin{aligned} v_z(x, y, z) &= \frac{\exp(ikz)}{ikz} \iint_{\infty} v(x_0, y_0, z_0) \exp\left\{\frac{i\pi}{\lambda z} [(x - x_0)^2 + (y - y_0)^2]\right\} dx_0 dy_0 = \\ &= \frac{\exp(ikz)}{ikz} \exp\left\{\frac{i\pi(x^2 + y^2)}{\lambda z}\right\} \iint_{\infty} v(x_0, y_0, z_0) \exp\left\{\frac{i\pi(x_0^2 + y_0^2)}{\lambda z}\right\} \exp\left\{-\frac{i2\pi(x_0 x + y_0 y)}{\lambda z}\right\} dx_0 dy_0 \end{aligned} \quad (3)$$

Such a transition will make it possible to use the well-known properties of the Fourier transform, and in the case of digital holography, to use the fast Fourier transform algorithms.

The intensity distribution recorded by the CCD matrix, which is an interference pattern, is, in fact, a digital hologram. If we apply the operation of the inverse Fresnel transform to such a



hologram, and in our case this action is reduced to the inverse Fourier transform, then we can see the reconstructed imaginary and real images of the original object and, also, the zero diffraction order. Analytically, this procedure can be described as follows

$$v(x_0, y_0, z_0) = \frac{\exp(ikz_0)}{ikz_0} \iint_{\infty} v_z(x, y, z) \exp\left\{-\frac{i\pi}{\lambda z} [(x-x_0)^2 + (y-y_0)^2]\right\} dx dy =$$

$$= \frac{\exp(ikz_0)}{ikz_0} \exp\left\{-\frac{i\pi(x_0^2 + y_0^2)}{\lambda z_0}\right\} \iint_{\infty} v_z(x, y, z) \exp\left\{-\frac{i\pi(x^2 + y^2)}{\lambda z}\right\} \exp\left\{\frac{i2\pi(x_0x + y_0y)}{\lambda z}\right\} dx dy$$

(4)

If we are talking about the intensity distribution in the plane of the hologram or, in our case, in the plane of the CCD matrix, then the distribution of the intensity of the light field, which, in fact, is an interference pattern recorded by the CCD matrix, has the form

$$I(x, y) = |v(x, y)|^2.$$

(5)

Based on formula (1), we can write for the intensity

$$I(x, y) = I_u(x, y), \text{ where } I_u(x, y) = |u(x, y)|^2.$$

(6)

Let us consider the applicability of these formulas in digital holographic interferometry.

Let's change the phase of the object wave to $\Delta\theta(x_0, y_0)$. Then the object wave in the input plane has the form $u'(x_0, y_0) = a \exp[i\theta(x_0, y_0) + i\Delta\theta(x_0, y_0)]$. In this case, the object wave in the hologram recording plane has the form

If we consider the case of two exposures used in holographic interferometry, then mathematically this means the restoration of the sum of two waves - the initial one $u(x, y)$ and with a changed phase $u'(x, y)$. Since the interference pattern is recorded in the form of an intensity distribution, then

$$I(x, y) = [u(x, y) + u'(x, y)][u(x, y) + u'(x, y)]^* =$$

$$= \{a \exp[-i\theta(x, y)] + a \exp[-i\theta(x, y)] \exp[-i\Delta\theta(x, y)]\} \times$$

$$\times \{a \exp[i\theta(x, y)] + a \exp[i\theta(x, y)] \exp[i\Delta\theta(x, y)]\} =$$

$$= 2a^2 + a^2 \{\exp[i\Delta\theta(x, y)] + \exp[-i\Delta\theta(x, y)]\}$$

(8)

Let us use Euler's formula to pass from the complex form of recording the intensity to the real one

$$I(x, y) = 2a^2 + 2a^2 \cos[\Delta\theta].$$

(9)

Based on relations (6), expression (9) can be represented as

$$I_{\Sigma} = I_u(x, y) B_{\Sigma} \{1 + \cos[\Delta\theta(x, y)]\},$$

(10)

where B_{Σ} is some real coefficient describing the total, i.e., the total brightness of the image, I_{Σ} is the image of the reconstructed object, modulated by interference fringes, in the case of usual, not improved sensitivity.

If it is necessary to find separately the value of the intensity I_{Σ} and the corresponding value of the phase shift $\Delta\theta(x, y)$, you can use the following relations

$$I_u(x_0, y_0) = |F^{-1}\{v(x, y)\}|^2,$$

(11)



$$\exp[i\Delta\theta(x_0, y_0)] = \frac{F^{-1}\{v'(x, y)\}}{F^{-1}\{v(x, y)\}}.$$

(12)

Here F^{-1} means the operator of the inverse Fourier transform, and F , accordingly, describes the operator of the direct Fourier transform.

As can be seen from relations (11) and (12), they contain essential information about the amplitudes and phase characteristics of the waves that form the interference pattern.

Consider the possibility of increasing the sensitivity of the interference system.

The presence of digital information about the object and reference waves, about the interference pattern fixed by the CCD matrix allows performing various mathematical operations on these data, after which they can again be displayed on the screen, i.e., visualized. These operations can be carried out both in the sections from the object to the output plane, and at the output of the system, directly with the intensity distribution, which is fixed by the CCD matrix. These are operations such as digital filtering, convolutions, etc.

Let's carry out the sequence of the following mathematical operations on the data obtained at the output of our interferometric system:

1. Let's perform the operation of subtracting the light fields in the output plane from the original object and the object with a changed phase. We will restore the obtained result, that is, in our case it is the operation of the inverse Fourier transform, and then we will represent the result in the form of the intensity distribution for the difference of the light fields.

$$\begin{aligned} \Delta v(x_0, y_0, z_0) &= \\ &= \frac{\exp(ikz_0)}{ikz_0} \iint_{\infty} [v'_z(x, y, z) - v_z(x, y, z)] \exp\left\{-\frac{i\pi}{\lambda z} [(x-x_0)^2 + (y-y_0)^2]\right\} dx dy = \\ &= \frac{\exp(ikz_0)}{ikz_0} \exp\left\{-\frac{i\pi(x_0^2 + y_0^2)}{\lambda z_0}\right\} \times \\ &\times \iint_{\infty} [v'_z(x, y, z) - v_z(x, y, z)] \exp\left\{-\frac{i\pi(x^2 + y^2)}{\lambda z}\right\} \exp\left\{\frac{i2\pi(x_0x + y_0y)}{\lambda z}\right\} dx dy \end{aligned}$$

(13)

We find the intensity distribution in the form of the relation

$$I_1(x_0, y_0) = |\Delta v(x_0, y_0, z_0)|^2 = [\Delta v(x_0, y_0, z_0)][\Delta v(x_0, y_0, z_0)]^*,$$

(14)

since in the general case $\Delta v(x_0, y_0, z_0)$ is a complex expression. For the exit plane, expression (14) can be represented, by analogy with expression (10), in the following form

$$I_1(x, y) = I_u(x, y) B_{\Sigma} \{1 - \cos[\Delta\theta(x, y)]\}.$$

(15)

2. Subtract expression (15) from relation (10). The resulting result is raised to the second power



$$\begin{aligned}
I_2 &= [I_\Sigma(x, y) - I_1(x, y)]^2 = \\
&= \{I_u(x, y)B_\Sigma \{1 + \cos[\Delta\theta(x, y)]\} - I_u(x, y)B_\Sigma \{1 - \cos[\Delta\theta(x, y)]\}\}^2 = \\
&= \{2I_u(x, y)B_\Sigma \cos[\Delta\theta(x, y)]\}^2 = \\
&= I_u^2(x, y)B_\Sigma^2 \{2\cos[\Delta\theta(x, y)]\}^2 = \\
&= 2I_u^2(x, y)B_\Sigma^2 \{1 + \cos[2\Delta\theta(x, y)]\} = \\
&= I_u^2(x, y) \frac{B_\Sigma^2}{2} \{1 + \cos[2\Delta\theta(x, y)]\} = I_u^2(x, y)B_2 \{1 + \cos[2\Delta\theta(x, y)]\}
\end{aligned}$$

(16)

It can be seen from relation (16) that the sequence of performed operations doubles the phase shift between the original and distorted waves, which also doubles the frequency of interference fringes in the output plane. That is, the sensitivity of the holographic interferometer is doubled. The interferometer can capture inhomogeneities in the phase medium, the magnitude of which is two times less than before the considered mathematical operations.

Let's multiply relations (10) and (15)

$$\begin{aligned}
I_3 &= I_\Sigma(x, y)I_1(x, y) (I_u(x, y)B_\Sigma \{1 + \cos[\Delta\theta(x, y)]\}) * \\
&* (I_u(x, y)B_\Sigma \{1 - \cos[\Delta\theta(x, y)]\}) = \\
&= I_u^2(x, y)B_\Sigma^2 \{1 - \cos^2[\Delta\theta(x, y)]\} I_u^2(x, y) \frac{B_\Sigma^2}{2} \{1 - \cos[2\Delta\theta(x, y)]\}.
\end{aligned}$$

(17)

We will carry out a sequence of actions similar to those that were carried out to obtain relation (16), that is, subtract relation (17) from expression (16), and square the result obtained

$$\begin{aligned}
I_4 &= [I_2(x, y) - I_3(x, y)]^2 = \\
&= (I_u^2(x, y) \frac{B_\Sigma^2}{2} \{1 + \cos[2\Delta\theta(x, y)]\} - I_u^2(x, y) \frac{B_\Sigma^2}{2} \{1 - \cos[2\Delta\theta(x, y)]\})^2 = \\
&= I_u^4(x, y) \frac{B_\Sigma^4}{4} 4 \cos^2[2\Delta\theta(x, y)] I_u^4(x, y) \frac{B_\Sigma^4}{2} \{1 + \cos[4\Delta\theta(x, y)]\}.
\end{aligned}$$

(18)

If relation (16) doubles the sensitivity of the interference system, then from expression (18) it can be seen that in this case the phase difference between the reference, i.e., the original object wave, and the wave with a changed phase increases four times. As a result, the spatial frequency of the interference pattern localized on the reconstructed real image of the object under study increases fourfold.

In a generalized form, the proposed for I_2 and I_4 scheme for increasing the sensitivity of the holographic interference system can be represented as follows

$$I_{2^m} = I_u^{2^m} \frac{B_\Sigma^{2^m}}{2} \{1 + \cos[2^m \Delta\theta(x, y)]\}.$$

(19)

Here $m = 0, 1, 2, 3, \dots$

It can be seen from relation (19) that the numerical increase in the sensitivity of the holographic interferometer is limited, in fact, only by possible speckle noise, which can worsen the contrast of the fringes of the interference pattern.

Conclusion



It is shown that the presence of digital information about the object and reference waves, about the interference pattern recorded by the CCD matrix allows performing various mathematical operations on these data, after which they can again be displayed on the screen, i.e., visualized. These operations can be carried out both in the sections from the object to the output plane, and at the output of the system, directly with the intensity distribution, which is fixed by the CCD matrix.

A sequence of numerical operations is proposed, which makes it possible to double the sensitivity of the holographic system. It is shown that the proposed scheme for increasing the sensitivity of a holographic interferometer can be generalized to the case of increasing the sensitivity by a factor of 2^m , where $m = 0, 1, 2, 3, \dots$. It is also shown that the increase in sensitivity according to this scheme is limited only by possible speckle noise, which can worsen the contrast of the fringes of the interference pattern.

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