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BOLOTBEK T., MIRLANOV K.M., TELIN A.Y., CHUKANOV E.S., TALGATOV A.T.
[1]Kyrgyz State University of Construction, Transport and Architecture named after N. Isanov (KSUCTA), Bishkek, Kyrgyz Republic
temir.b.amir@gmail.com

БОЛОТБЕК Т., МИРЛАНОВ К.М., ТЕЛИН А.Ю., ЧУКАНОВ Э.С., ТАЛГАТОВ А.Т.
[1]Кыргызский государственный университет строительства, транспорта и архитектуры им. Н.Исанова (КГУСТА), Бишкек, Кыргызская Республика

SPECTRAL METHODS FOR DETERMINING THE SEISMIC FORCES OF BUILDINGS

СПЕКТРАЛЬНЫЕ МЕТОДЫ ОПРЕДЕЛЕНИЯ СЕЙСМИЧЕСКИХ СИЛ ЗДАНИЙ

Макалада спектрдик ийри сызык боюнча имараттарга жана курулмаларга сейсмикалык таасирлерди эсептөөнүн ыкмалары, конструкциялардын ташуучу бөлүктөрүнүн динамикалык эсептөө схемаларын түзүү, инерциялык реакциялар жана курулуш конструкцияларын которуу, сейсмикалык күчтүн таасиринен улам имараттардын өзүнүн жана аргасыз термелүүлөрү каралат.
Өзөк сөздөр: *курулуш конструкциялары, конструкциянын ташуучу бөлүгү, сейсмикалык таасир, сызыктуу термелүүлөр, инерциялык реакция, деформация, көтөрүү жөндөмдүүлүгү, серпилгичтик, катуулук.*

В статье рассматриваются методы расчета сейсмических воздействий на здания и сооружения по спектральной кривой, создание динамической расчетной схемы несущих элементов конструкций, инерционные реакции и перемещения строительных конструкций, собственные и вынужденные колебания зданий при действии сейсмических сил.
Ключевые слова: *строительная конструкция, несущий элемент конструкции, сейсмическое воздействие, линейное колебание, инерционная реакция, деформация, несущая способность, упругость, жесткость.*

The article discusses methods for calculating seismic effects on buildings and structures along the spectral curve, creating a dynamic calculation scheme for load-bearing structural elements, inertial reactions and displacements of building structures, natural and forced vibrations of buildings under the effect of seismic forces.
Keywords: *building structure, load-bearing structural element, seismic action, linear vibration, inertial reaction, deformation, bearing capacity, elasticity, rigidity.*

Relevance. With the current level of development of computer technologies and software applications, calculations for the periods and modes of natural vibrations of building structures are relatively easy to solve, for these purposes there is a wide range of software products such as Autodesk Revit, Autodesk Robot Structural Analysis, Lira, etc. The main problems are of fundamental nature and based on the difficulty of determining seismic forces. The function *Wo (t) = Y (t)*, the model of rock fluctuations in the calculation formulas, is unstable and does not provide an accurate analytical explanation. Seismic forces are defined in the same vein.

Research Purpose. Perform numerical modeling of buildings and structures on the spectral curve.

Methodology. The following methods of structural mechanics were used in the work: a spectral method for determining seismic forces, the method of concentrated deformations, the method of boundary conditions, methods of graphic statics, methods of statically determinate and statically indeterminate systems, fundamentals of the seismic vibrations theory.

Research Results. Static theory has been primarily used in retrospect. The deformation of building structures according to this theory was insignificant and the acceleration of points with the acceleration of the ground was synchronized. The highest indicators of seismic inertia are calculated from the maximum ground acceleration [1]:

$$S_k = \max\left|S_k(t)\right| = m_k \max W_0 = \frac{\max W_0}{g} Q_k = K_c Q_k . \qquad (1)$$

Here Q_k – is the weight of the concentrated load at the point k.

Current methods for determining seismic loads on buildings and structures are based on a dynamic approach to the problem and they use the theory of seismic vibrations. One of these methods is the calculation based on real records of earthquakes that have occurred. Seismic forces are calculated by numerically solving differential oscillation equations, and the instrumental accelerogram of one of the last strong earthquakes in a real and specific area is taken as a *Y(t)* function. Calculations are performed using Robot Structural Analysis, Lira or any other available on the market software. Among other things, the disadvantage of this method is the complexity of choosing an accelerogram that is sufficiently suitable for this particular case.

An alternative method is based on the probabilistic interpretation of the seismic vibrations problem [2, 3, 4]. This allows deformations to be determined based on the properties of seismic forces or properties of fundamental vibrations of the environment. The possibilities of this method are limited by the lack of basic knowledge about the probabilistic properties of rock vibrations.

The spectral numerical model is used to determine the seismic forces. It does not give an accurate description of the seismic forces fluctuations in different ranges of temporal data. The method approximates seismic forces for individual normal components of buildings and structures oscillatory process.

Then we derive the mathematical formulas of the spectral method. The absolute value of the maximum seismic forces during normal vibrations is:

$$S_{ik} = \max_t \left|S_{ik}(t)\right| m_k X_{ik} D_i \left(\frac{2\pi}{T_i} \max_t \left|I_i(t)\right|\right)$$

$$(k = 1,2,\ldots,n)$$

We enter the notation as follows:

$$C_W(T) = \max_t \left|W(t,T,\gamma)\right| . \qquad (2)$$

This function, which expresses the greatest accelerations of the generator depending on its own period, is called the acceleration spectrum. Its graphical signature is a spectral acceleration curve. The displacement and velocity spectra of the oscillator are calculated in an identical way.

While taking into account the notation (1), the previous expression of the maximum inertial forces will be rewritten in the following form:

$$S_{ik} = m_k X_{ik} D_i C_W(T_i)$$
$$(k = 1,2,\ldots,n) \qquad (3)$$

Thus, the maximum inertial forces of the individual normal components of seismic vibrations are expressed using the acceleration spectrum. In this vein, the acceleration spectrum can be considered as the main engineering and technical characteristic of the seismic impact.

Now we will replace the masses m_k in expression (3) with the concentrated weights Q_k according to the relation

$$m_k = \frac{Q_k}{g}, \tag{4}$$

where g is the gravitational force. In addition, we will introduce the following notation:

$$\eta_{ik} = X_{ik} D_i; \tag{5}$$

$$\frac{C_W(T_i)}{g} = K_c \beta_i. \tag{6}$$

Then expression (3) is written as follows:

$$S_{ik} = K_c \beta_i \eta_{ik} Q_k \quad (k = 1,2,\ldots,n) \tag{7}$$

Formula (7) is the key calculation formula for existing building codes and regulations. It determines the concentrated seismic forces of a discrete contour corresponding to the i-th normal component. In formula (7), βi is called the dynamic coefficient, and $\acute{\eta} ik$ – is the shape coefficient, Kc – is the seismicity coefficient. Let us include the value of Di in formula (5), replacing the weights mk in it in proportion to the values of Qk. Then for the coefficient of the form we obtain the following expression:

$$\eta_{ik} = X_{ik} \frac{\sum\limits_{v=1}^{n} Q_v X_{iv}}{\sum\limits_{v=1}^{n} Q_v X_i^2}. \tag{8}$$

As shown, the indicated coefficient is derived from the natural form only and the localization of the load Qk in the marked design scheme.

It follows from formula (6) that $Kc\beta i$ is the spectrum of accelerations, expressed in fractions of the gravitational constant acceleration. The Kc values in the norms are the same as in the static theory. The desired value of the seismicity coefficient should be considered as a key factor. The properties of the seismic response spectrum are reflected in βi coefficient. Its values are based on the spectral curve.

Based on the spectral curves, it is possible to determine the inertial forces for systems with distributed parameters. The maximum intensity of the distributed inertial forces of the i-th normal component can be represented as [5, 1]:

$$S_i(x) = \max_t \left| S_i(x,t) \right| = K_c \beta_i \eta_i(x) q(x) . \quad (9)$$

Here *q(x)* is the intensity of the vertical force configuration. The eigenmode shape coefficient in this case is determined by the following formula

$$\eta_i(x) = X_i(x) D_i = X_i(x) \frac{\int_0^h q(x) X_i(x) dx}{\int_0^h q(x) X_i^2(x) dx} , \quad (10)$$

where h is the height of the structure under consideration.

In this regard, the response spectrum curves describe the largest inertial forces for the seismic wave components. To compensate for this shortcoming of the spectral method, it is reasonable to resort to additional assumptions about the phase relationship of individual normal components [6, 7, 1].

The probabilistic calculation takes into account the first few normal components of seismic vibrations. For each of them, the maximum seismic loads are calculated using formulas (3) or (9). In addition, the seismic forces for each component are considered as an independent static load, and the seismic forces in the section *Ni* corresponding to the individual components are calculated in the usual way. Their maximum values for this particular section are denoted by *Nmax*. The design force in the cross section is defined as the root-mean-square value of these forces, all *Ni*, except for its maximum value, are entered with a factor of 0.8.

From the mathematical expressions described above, it can be seen that the response spectra of individual earthquakes can be determined from instrumental accelerograms. They can also be obtained from alternative seismometers that directly record the maximum accelerations of oscillators with different periods and shapes [8, 9, 10]. The calculated response spectrum can be obtained by cumulative generalization of the spectra of individual earthquakes characteristic of a particular region. We assume that with the accumulation of information data from seismometers, it will be possible to construct refined spectral curves for each individual region.

In the building codes of some countries, the calculated spectral curves take into account the influence of the soil conditions of a particular area [8].

The spectral method for determining seismic loads is based on the fact that, depending on the periods and modes of natural oscillations of building structures, calculated according to their dynamic design scheme, seismic loads for individual natural modes and periods of oscillations are determined using a spectral curve. For these loads, considered as static loads, the seismic forces are determined from individual natural modes and oscillation periods. Next, the numerical values of seismic loads are calculated, due to the entire complex of accepted normal components of seismic vibrations. The process of calculating seismic loads based on the dynamic method is shown below according to their parameters in sequential order.

In order to create a design model of a discrete type, the distributed vertical load in the supporting structures is shown in separate sections and in the form of concentrated loads used in the centers of gravity of the corresponding structural elements. But it is difficult to use for unique buildings and structures, and for many man-made structures. In practice, the concentrated element load is calculated as the total load on the corresponding structural elements. The load should be applied when the loads on reinforced concrete structures are high and earthquakes are close to the nature of the prevailing seismic vibrations. In addition, the authors believe that the results are based on the dynamic calculation scheme shown in Figure 1.

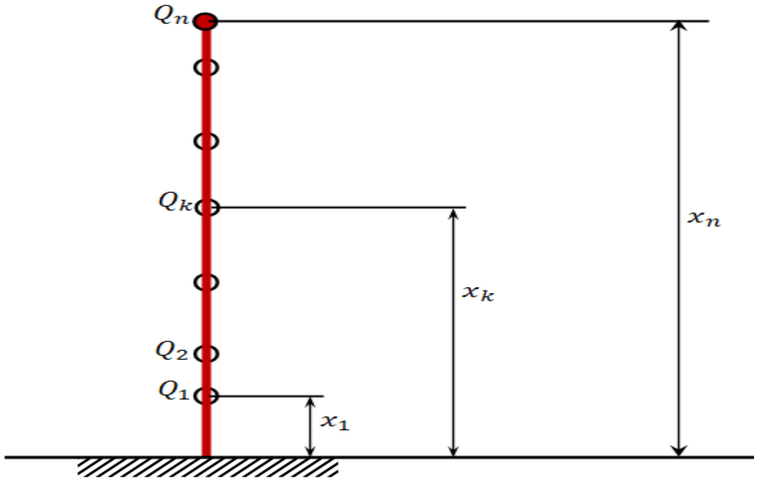

Fig. 1. Numerical model with *n* degrees of freedom

The power *Qk* is determined by the standard mass corresponding to building structures. The hydrostatic E_p of groundwater drainage is the basis of the surrounding particles and does not affect the mass. In this regard, some design solutions do not take into account the effect of hydrostatic pressure. During calculations, ongoing dynamic calculations can be calculated using the same method as building structures, which can be transferred to the overall structure. When calculating the concentrated pressure on a dynamic load element from an automatic switchgear, this effect is fixed using an additional factor of 0.8.

Single displacements are determined according to the rules of structural mechanics, while taking account the actual rigidity of the elements of building structures into account. It should be taken into account that load-bearing elements of structures that are not involved in strength calculations often increase the rigidity of the structure to a significant extent, which will inevitably lead to an increase in seismic inertial forces. Thus, for example, the excessive horizontal rigidity of spans of bridge structures in the transverse direction, in addition to horizontal ties, depends significantly on the road surface. Not taking into account these structural elements can give a result that does not exist in fact, which must be taken into account when calculating single displacements. In some cases, it is recommended to take into account the elastic-plastic characteristics of foundation structures.

The higher the rigidity of the load-bearing structural elements and the identical its scheme to a system with one degree of freedom, the higher the components (overtones) of seismic vibrations. Determining the number of components to be calculated, the authors recommend determining the intrinsic step period *T1*, which is a measure of the dynamic stiffness of building structures.

For *T1>1/2 s*, the first three normal components should be taken into account (*i = 1,2,3*). If flexible structures such as suspension bridges, it becomes necessary to consider a larger number of components (up to five). For structures with a period *T1<1/2 s*, the norms allow taking into account only 1 component of seismic vibrations in the calculation.

To redefine *Qk* and unit displacements *δkv* of concentrated masses, it is necessary to calculate masses with concentration *mk = Qk/g* and form a matrix of the following form:

$$\left[m_k \delta_{kv}\right]_{kv}^{1,n} = \begin{matrix} m_1\delta_{11} & m_2\delta_{12} & \dots & m_n\delta_{1n} \\ m_1\delta_{21} & m_2\delta_{22} & \dots & m_n\delta_{2n} \\ \dots & \dots & \dots & \dots \\ m_1\delta_{n1} & m_2\delta_{n2} & \dots & m_n\delta_{nn} \end{matrix} \qquad (11)$$

Hereafter, for this matrix, it is necessary to calculate the first exponents of the eigenvalues *λi* and the coordinates of the corresponding eigenvectors *Xik*. Secondly, it determines the wave height coefficients, that is, the coordinates of the corresponding modes natural vibrations at the points of application of the concentrated forces *Xik =CXi (xk)*. Their duration is expressed as eigenvalues

$$T_i = 2\pi\sqrt{\lambda_i}\,. \tag{12}$$

The real problem here is to calculate the eigenvalues and eigenvectors. It is well known that the characteristic equation is used to determine the eigenvalues:

$$\begin{vmatrix} m_1\delta_{11}-\lambda & m_2\delta_{12} & \dots & m_n\delta_{1n} \\ m_1\delta_{21} & m_2\delta_{22}-\lambda & \dots & m_n\delta_{2n} \\ \dots & \dots & \dots & \dots \\ m_1\delta_{n1} & m_2\delta_{n2} & \dots & m_n\delta_{nn}-\lambda \end{vmatrix} = 0. \tag{13}$$

The eigenvectors are calculated from the following system of linear homogeneous mathematical equations:

$$\left.\begin{aligned} &(m_1\delta_{11}-\lambda_i)X_{i1} + m_2\delta_{12}X_{i2} + \dots + m_n\delta_{1n}X_{in} = 0 \\ &m_1\delta_{21}X_{i1} + (m_2\delta_{22}-\lambda_i)X_{i2} + \dots + m_n\delta_{2n}X_{in} = 0 \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &m_1\delta_{n1}X_{i1} + m_2\delta_{n2}X_{i2} + \dots + (m_n\delta_{nn}-\lambda_i)X_{in} = 0 \end{aligned}\right\}. \tag{14}$$
$$i = 1,2,\dots,n$$

The eigenvalue $\lambda = m\delta$ and, accordingly, the oscillation values are calculated by the following formula:

$$T = 2\pi\sqrt{m\delta} = \frac{2\pi}{\sqrt{g}}\sqrt{Q\delta}\,. \tag{15}$$

where *m, Q* are the weight and load of the system, δ is the unit horizontal displacement of their pinching point.

For a system with two degrees of freedom, visualized in Fig. 2 we derive the following quadratic equation

$$\lambda_{1,2} = \frac{2m_1m_2\left(\delta_{11}\delta_{22}-\delta_{12}^2\right)}{m_1\delta_{11}+m_2\delta_{22} \mp \sqrt{(m_1\delta_{11}+m_2\delta_{22})^2 - 4m_1m_2\left(\delta_{11}\delta_{22}-\delta_{12}^2\right)}}\,. \tag{16}$$

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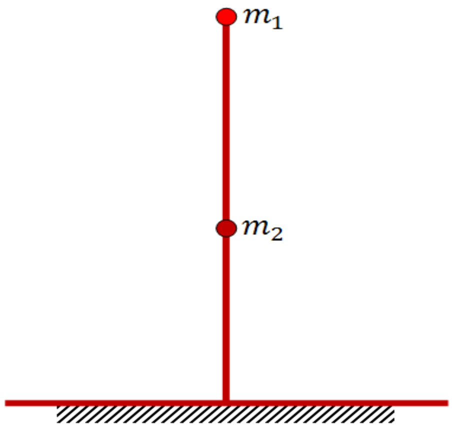

Fig. 2. Numerical model with 2 degrees of freedom

According to equation (14), the amplitude-frequency coefficients can be written as follows:

$$X_{i1} = 1; X_{i2} = \frac{\lambda_i - m_1\delta_{11}}{m_2\delta_{12}} \tag{17}$$

$$(i = 1,2)$$

Most likely, in some cases, only the base form and the corresponding definition of *T1* can be defined. Without software applications, it cannot be used by the method of successive approximations [28] or the spectral curve method. Approximation to reality and the form of the energy method can be obtained on the basis of the energy method [2, 3, 4]. The concentrated masses *Qq* are applied to the scaled circuit in the horizontal circuit and determine the point of application, based on this, the following is true:

$$y_k = \sum_{v=1}^{n} Q_v \delta_{kv} \tag{18}$$

$$(k = 1,2,\ldots,n)$$

These displacements are taken as approximate values of the ordinates of the form of the main vibration (*yk=X1k*).

Pitch period:

$$T_1 = \frac{2\pi}{\sqrt{g}} \sqrt{\frac{\sum_{k=1}^{n} Q_k y_k^2}{\sum_{k=1}^{n} Q_k y_k}}\,. \tag{19}$$

It should be noted that the actual deformation properties of building structures and their foundations in the calculations are always reflected with a high degree of approximation, which introduces an error in determining the proper periods. Therefore, we recommend evaluating the value of proper periods obtained as a result of calculation in accordance with full-scale experiments of similar building structures.

Concentrated seismic loads correspond to the *i*-th natural mode:

$$S_{ik} = K_c \beta_i \eta_{ik} Q_k \quad (20)$$
$$(k = 1,2,\ldots,n)$$

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Recall that S_{ik} is a concentrated seismic load generated by the weight Q_k and applied to the point *k*. The seismicity coefficient K_c depends on the analytical seismicity of the structure given in Table 1 [1].

Table 1 – Analytical seismicity of the structure [1]

Analytical seismicity, MSK points	7	8	9
K_c	0,025	0,0 5	0, 1
A; A=K_c*2	0,05	0,1	0, 2

The coefficients *ήik* are calculated by expression (8). The dynamic coefficient *βi* is determined by the natural oscillation period *Ti* of each specific seismic action. The response spectrum curve of the dynamic coefficient, adopted in building codes, is visualized in Fig. 3. The values of *β* are bound by the conditions *β<3; β>0.8*, the intermediate part of the curve is described by the following formula

$$\beta_i = \frac{1}{T_i}. \quad (21)$$

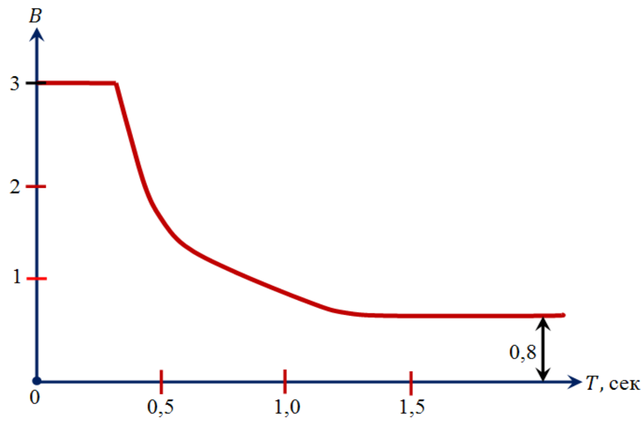

Fig. 3. Graph of the coefficient of dynamics

According to building codes, the design seismic forces in any section are determined by the following formula

$$N_p = \sqrt{N_{\max}^2 + 0{,}5 \sum N_i^2}, \quad (22)$$

where N_i is the force in the section from the seismic forces of the *i*-th normal component; N_{max} is maximum force.

The practice of applying the above theoretical calculations and the findings can be implemented using the example of [11, 12].

Conclusions. The seismic forces were calculated for individual normal components. For this purpose, the set of seismic loads *Sik* corresponding to each *i*-th component is considered as an independent static load. For these forces, seismic loads in the determined sections of the building structure are calculated in a classical way. Thus, each calculated section has its own amount of seismic forces *Ni* corresponding to different natural forms. Here *r* is the number normal components taken into account in the calculation, as a rule, r = *3*.

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