ШАБЛОНЫ ЧИСЛЕННОГО РЕШЕНИЯ ЗАДАЧ ТЕОРИИ УПРУГОСТИ В СРЕДЕ EXCEL НА ОСНОВЕ ДИСКРЕТНОЙ МОДЕЛИ

PATTERNS OF NUMERICAL SOLUTION OF PROBLEMS OF THE THEORY OF ELASTICITY IN EXCEL BASED ON A DISCRETE MODEL

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Questions of calculating elastic deformation in the environment of Excel are considered, based on the use of discrete model [1,3]. The regularity of change of movements and the stretching of deformation in the final strip squeezed by the concentrated forces are established.

1.   Initial considerations. It is difficult to solve the tasks of theory of elasticity, when bodies of the final size are considered. The results can be obtained by numerical calculations. The choice of the one or another settlement scheme is utmost important (templates) [2]. The template with sexatomic model of a molecule is considered, of which the solid body consists [1,3]. Different application program packages are used for the numerical solutions of tasks of the theory of elasticity . The simplest application is the built-in functions of an office application MS Excel. The realization of such opportunities has the scientific and practical importance.

Proceeding from this, given work analyses the problem of numerical calculation of a beam in the elastic strip (in case of flat deformation), squeezed by the concentrated symmetric forces (Fig.1).

The elastic beam with width 2h and lengths 2L, compressed in the middle by two cross concentrated forces F is given. It is required to define the greatest stretching deformations at various values of strength F to establish nature of the change.
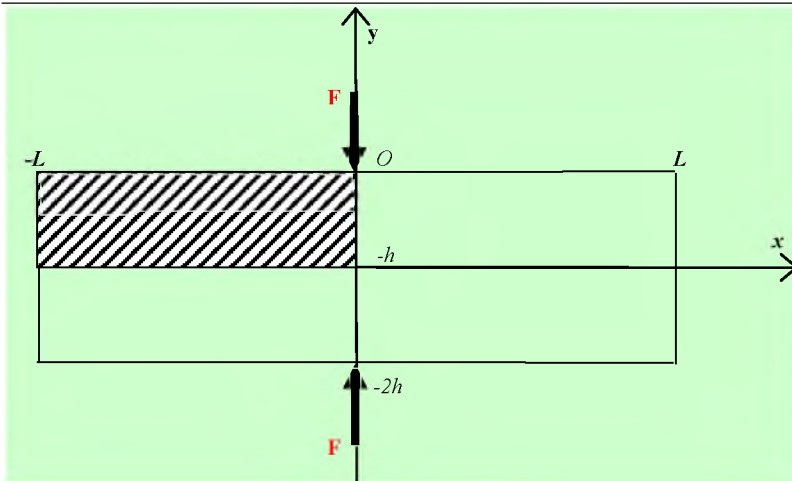
Fig.1 Settlement scheme of loading of a strip.

Taking into account the symmetry, we will further consider 1/4 part of a strip (on fig.1). In this case, horizontal (U) and vertical (V) movement of intersection points x=0, x=±ρ, and also x=± λ (λ=2ρ) under y ∈ (0; h), are defined by the following ratios:

$$U(0,y)=0 ; \quad U(+\rho,y)=-U(-\rho,y) ; \quad U(+\lambda,y)=-U(-\lambda,y) ;$$
$$V(+\rho,y)=V(-\rho,y) \quad \text{and} \quad V(+\lambda,y)=V(-\lambda,y) \tag{1}$$

Movements of intersection points y=-h, y=±ρ and y=± λ at x ∈ (0; -l)   are subjected to the conditions:

$$V(x,-h)=0; \quad V(x,-h-\rho)=-V(x,-h+\rho);$$
$$V(x,-h-\lambda)=-V(x,-h+\lambda);$$
$$U(x,-h-\rho)=U(x,-h+\rho) \quad \text{and} \quad U(x,-h-\lambda)=U(x,-h+\lambda) \tag{2}$$

For the numerical solution of an objective, the considered elastic strip is modelled using the discrete environment in the form of set "molecules", connected with each other through general "atoms". The separate molecule consists of 6 "atoms" (fig.2).
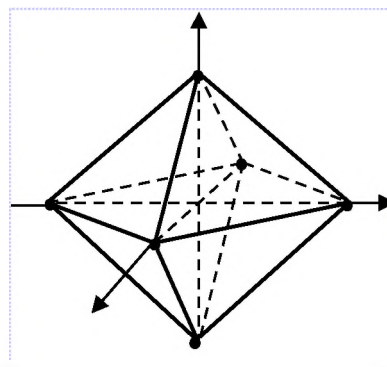

Fig.2. 6-"nuclear" model of elementary volume of an elastic body.

"Atoms of a molecule" of an elastic body interact with each other defined by the linear function, with coefficient of interaction of the "atoms" located at distance λ. This linear function is expressed as $K_1 = \mu\lambda^2/4$, and for "atoms" located at distance $\lambda/\sqrt{2}$ is $K_2 = \sqrt{2}\,\mu\lambda^2/2$. Where μ is rigidity modulus, characterizing elastic properties of a beam.

Characteristics of such a model of an elastic body (specified by coefficients) were defined by the laws of central interaction (i.e. attraction or repulsion) between two atoms and according to elastic behavior of macro uniform, isotropic body.

When using such a model, the considered area of a strip will have an appearance similar to the one shown in Fig.3.
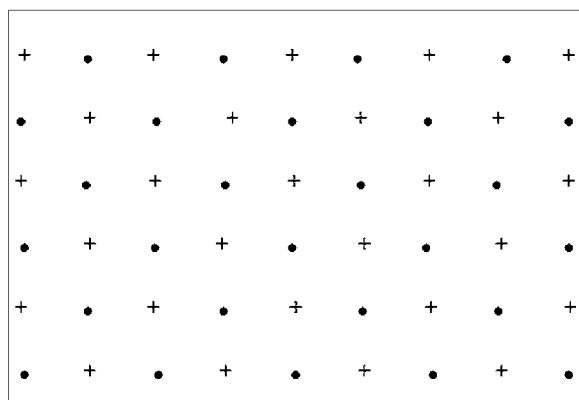

Fig.3. *The scheme of the allocated part of a strip on discrete model: "atoms (•)" located in the drawing plane, "atoms (+)" in the planes at distance $\pm\rho$ from the drawing plane.*

2.   The equations of an equilibrium condition of a strip in discrete model. In a body, each "atom" is generally a part of 6 "molecules" and interacts with 18 "atoms", comprising of these "molecules" (Fig .4). Movement of such an atom is connected with using the force on its volume, and also with movements of the atoms surrounding it. It has to meet a condition of an equilibrium state in balance with external (active) and internal (reactive) forces, depending on the relative positioning of the atoms. These equations are written in the form of coordinate axes projections [1]:

$X(x,y)=-\lambda\mu/4\{18U(x,y)-U(x-\lambda,y)-U(x+\lambda,y)-4[U(x-\rho,y)+U(x+\rho,y)]-2[U(x-\rho,y-\rho)+U(x-\rho,y+\rho)+U(x+\rho,y+\rho)+U(x-\rho,y-\rho)+W(x-\rho,y-\rho)-W(x-\rho,y+\rho)+W(x+\rho,y+\rho)-W(x+\rho,y-\rho)]\}$ **(3)**

$Y(x,y)=-\lambda\mu/4\{18W(x,y)-W(x-\lambda,y)-W(x+\lambda,y)-4[W(x-\rho,y)+W(x+\rho,y)]-2[W(x-\rho,y-\rho)+W(x-\rho,y+\rho)+U(x+\rho,y)+U(x-\rho,y)+U(x-\rho,y-\rho)-U(x-\rho,y+\rho)+U(x+\rho,y+\rho)-U(x+\rho,y-\rho)]\}$

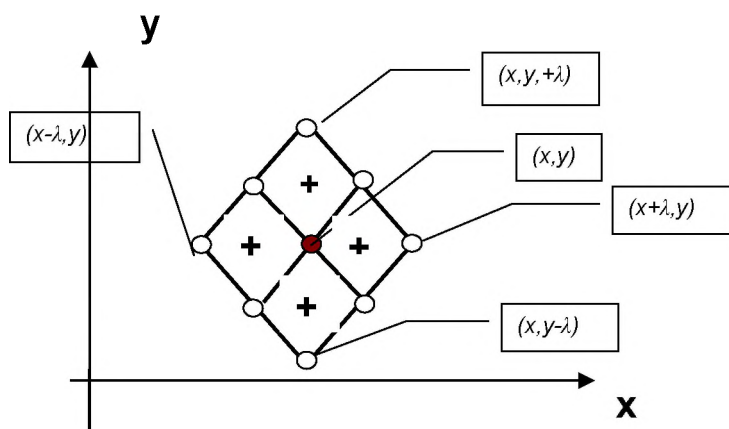Fig.4. *Communications of the atom having a full environment*

Where *(x, y)* are coordinates of "internal" "atom", *X (x, y)* and *Y (x, y)* are projections to axes *ox* and *oy* forces, acting on this atom. *U, W* are horizontal and vertical components of movement of the corresponding atoms.

The "atoms" located on a body surface with coordinates *(x,0)* have no full environment of molecules and they interact only with 6 "molecules" (Fig.5), The corresponding equations of balance is shown in Eq. 4.

$X(x,0)=-\lambda\mu/4\{14U(x,0)-U(x-\lambda,0)-U(x+\lambda,0)-4[U(x-\rho,0)+U(x+\rho,0)]-2[U(x-\rho,\rho)+U(x+\rho,-\rho\ \ +W(x-\rho,-\rho)-W(x+\rho,-\rho)]\}$

$Y(x,0)=-\lambda\mu/4\{9W(x,0)-W(x,-\lambda)-2[W(x-\rho,\rho)+W(x+\rho,-\rho)+U(x-\rho,-\rho)-U(x+\rho,-\rho)]-4W(x-\rho)\}$　　　　(4)

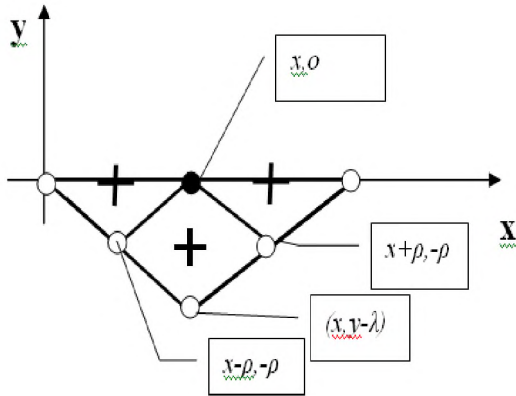*Fig.5. Interaction of superficial "atom"*

"Atoms" located close to the surface with coordinates *(x,- ρ)* also have no full environment of "molecules" (for this purpose there is no 1 atom (fig.6). The balance equations for them is shown in Eq. 5.

$X(x,y)=-\lambda\mu/4\{18U(x,y)-U(x-\lambda,y)-U(x+\lambda,y)-4[U(x-\rho,y)+U(x+\rho,y)]-2[U(x-\rho,y-\rho)++U(x-\rho,y+\rho)+U(x+\rho,y)+U(x-\rho,y)+W(x-\rho,y-\rho)-W(x-\rho,y+\rho)+W(x+\rho,y+\rho)-W(x+\rho,y-\rho)]\}$　　　(5)

$Y(x,-\rho)=-\lambda\mu/4\{17W(x,-\rho)-W(x,-3\rho)-4[w(x,\lambda)+W(x,0)]-2[W(x-\rho,-\lambda)+W(x-\rho,0)+W(x+\rho,0)+　W(x+\rho,-\lambda)+U(x-\rho,-\lambda)-U(x-\rho,0)+U(x+\rho,0)-U(x+\rho,-\lambda)]\}$
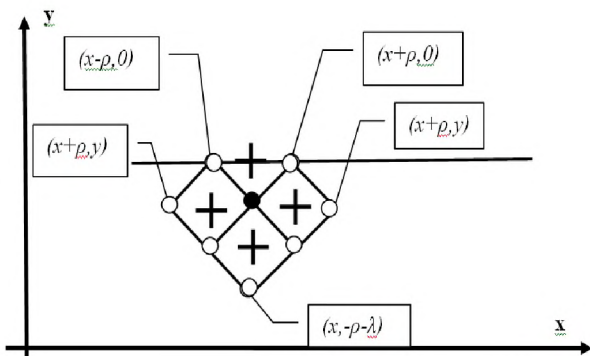

Рис.6

Fig.6. Interaction of atoms near a surface

For the atoms located on left most border *x= L, y ∈[0,-h]* and near this border (at distance ρ), we will apply a condition of absence of normal and shift deformations, which will be set by ratios given bellow.

$U(L,y)=U(L-\lambda,y);\ V(L,y)=V(L-\lambda,y)$

Similarly, for the atoms located in a near-surface layer *(x= L -ρ, y∈[0,-h])*, we apply the following conditions.

$U(L-\rho,y)=U(L-3\rho,y),\ V(L-\rho,y)=V(L-\rho,y).$

3. <u>The solution of this system of the equations</u> can be solved without difficulties, using applications such as MS Office Excel. For the numerical solution of task 2 following cases have been considered.

1) L=50λ, h=20λ,
2) L=25λ, h=11λ,

at the same time we compare the "atoms *(•)*" located in the drawing plane with the green colored

cells on the Excel sheet and to the "atoms **(+)**",

located in the planes at distance $\pm\rho$ from the drawing plane, are compared with white colored cells on the Excel sheet.

Considering parts of a strip in this application, it is possible to model the set of cells (Fig.3). Each cell represents certain "atom" and while the cell value represents movement of this "atom".
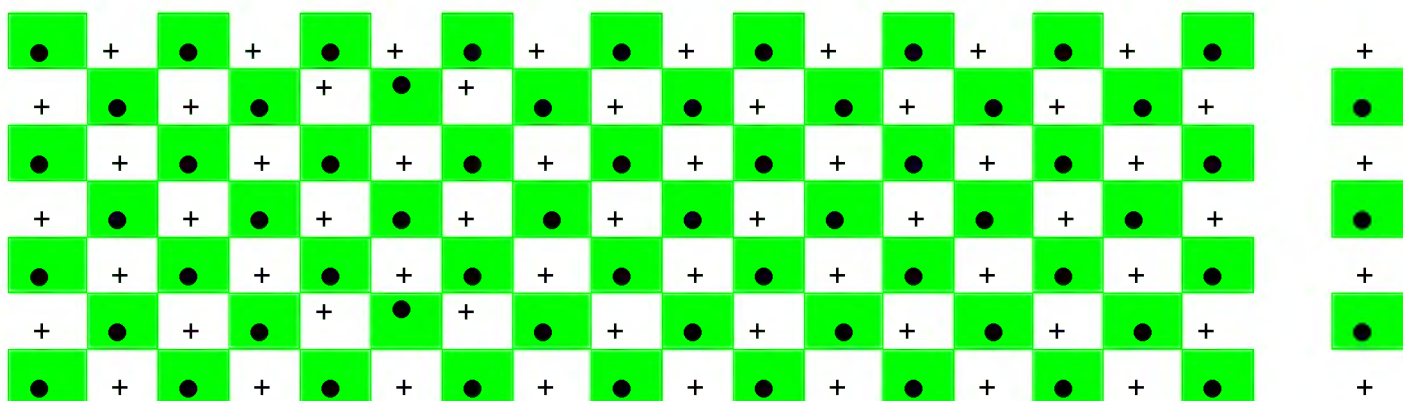

Fig.7. The settlement scheme in the Excel application

In each cell it is possible to calculate only one function, therefore for division two a component vertical*(V)* and horizontal *(U)* two Excel sheets are used. On the first sheet, horizontal component (*U*) of movements and on the second sheet vertical component *(V)* of movements were calculated. At the same time, the corresponding formulas are entered into each cell from Eq. 3, 4, 5 and correspond to boundary conditions.

Calculations of movements of "atoms" are made by the automatic function of the system of equations in the application, that uses iterative method.

For this purpose, after entering the corresponding formula in each cell, and in some free cell (for example H25) the value of force F is entered. Then the computing process can be started by

pressing F9 key. The results (numerical value of movements) are displayed in each cell as shown in Fig.8.

These results can be represented automatically in the form of charts (For example in Figure 7,8,9,10,11) The analysis of the computed results has shown that at distances $x > 5 \div 6\lambda$ as well as at $y < -(5 \div 6) \lambda$, the movements of atoms are considerably small, which can be neglected for practical purposes. Therefore, as basic model we have chosen a strip with sizes $L=12$, $h=8$.

	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB
1	-0,065	-0,061	-0,057	-0,052	-0,048	-0,044	-0,039	-0,035	-0,03	-0,027	-0,028	-0,04	-0,159	-0,159	-0,04	
2	-0,039	-0,036	-0,034	-0,031	-0,029	-0,026	-0,023	-0,021	-0,018	-0,017	-0,03	-0,041	-0,073	-0,073	-0,041	
3	-0,016	-0,015	-0,014	-0,013	-0,012	-0,011	-0,01	-0,009	-0,009	-0,01	-0,015	-0,021	-0,019	-0,019	-0,021	
4	-0,006	-0,006	-0,005	-0,005	-0,005	-0,005	-0,004	-0,005	-0,005	-0,006	-0,008	-0,006	-0,002	-0,002	-0,006	
5	-0,002	-0,002	-0,002	-0,002	-0,002	-0,002	-0,002	-0,003	-0,003	-0,003	-0,003	-8E-04	0,0004	0,0004	-8E-04	
6	-7E-04	-7E-04	-8E-04	-8E-04	-9E-04	-0,001	-0,001	-0,001	-0,002	-0,001	-5E-04	0,0002	0,0004	0,0004	0,0002	
7	-2E-04	-3E-04	-3E-04	-4E-04	-5E-04	-6E-04	-7E-04	-7E-04	-6E-04	-3E-04	-3E-05	0,0001	0,0001	0,0001	0,0001	
8	-9E-05	-1E-04	-2E-04	-2E-04	-3E-04	-3E-04	-3E-04	-3E-04	-2E-04	-7E-05	4E-06	1E-05	1E-05	1E-05	1E-05	
9	-4E-05	-6E-05	-8E-05	-1E-04	-1E-04	-1E-04	-1E-04	-1E-04	-5E-05	-2E-05	-1E-05	-1E-05	-2E-05	-2E-05	-1E-05	
10	-8E-06	-1E-05	-2E-05	-3E-05	-3E-05	-3E-05	-2E-05	-1E-05	-6E-06	-5E-06	-6E-06	-7E-06	-7E-06	-7E-06	-7E-06	
11	8E-06	1E-05	2E-05	3E-05	3E-05	3E-05	2E-05	1E-05	6E-06	5E-06	6E-06	7E-06	7E-06	7E-06	7E-06	
12	4E-05	6E-05	8E-05	0,0001	0,0001	0,0001	0,0001	1E-04	5E-05	2E-05	1E-05	1E-05	2E-05	2E-05	1E-05	
14	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0			

Fig.8. Type of cells after realization of computing procedure

Discussion of results

Calculations are carried out on force sizes of $4F/\mu\lambda$ = 1, 5,10,20,30 , 50. The separate results are represented in Fig. 9 and 10.
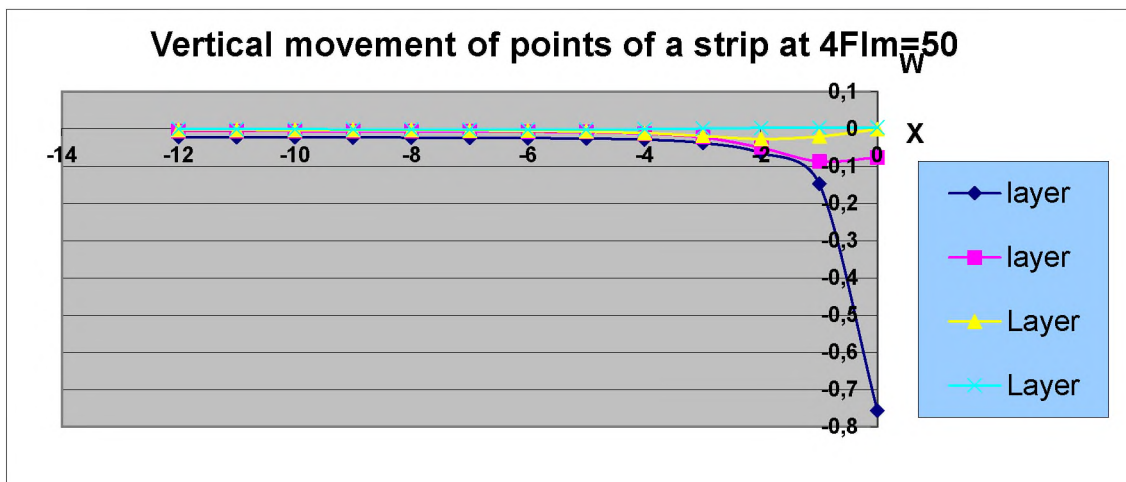

Fig.9. Charts of vertical movements

The row 1 (●——●) represents movement of superficial atoms, while the row 2 (◆——◆) movement of atoms in section x= -λ ,the row 3 ( ) movement of atoms in section y =-3ρ, and the row 4 ( )- movement of atoms in section -3λ. From Fig., it can be observed that the largest movements are of the "atoms" located on a beam surface. At long distances from a point of application of force these movements are insignificant and for simplification of the solution of a task we have chosen area of the size of x ∈ (0,-8); y ∈ (0,-8). In this area we see practically all range of changes of movements of atoms.

At all values of force **F** where the graph lines are similar, the change of vertical movements of atoms are identical. The greatest movement of atoms happen in the field of influence of the applied force.

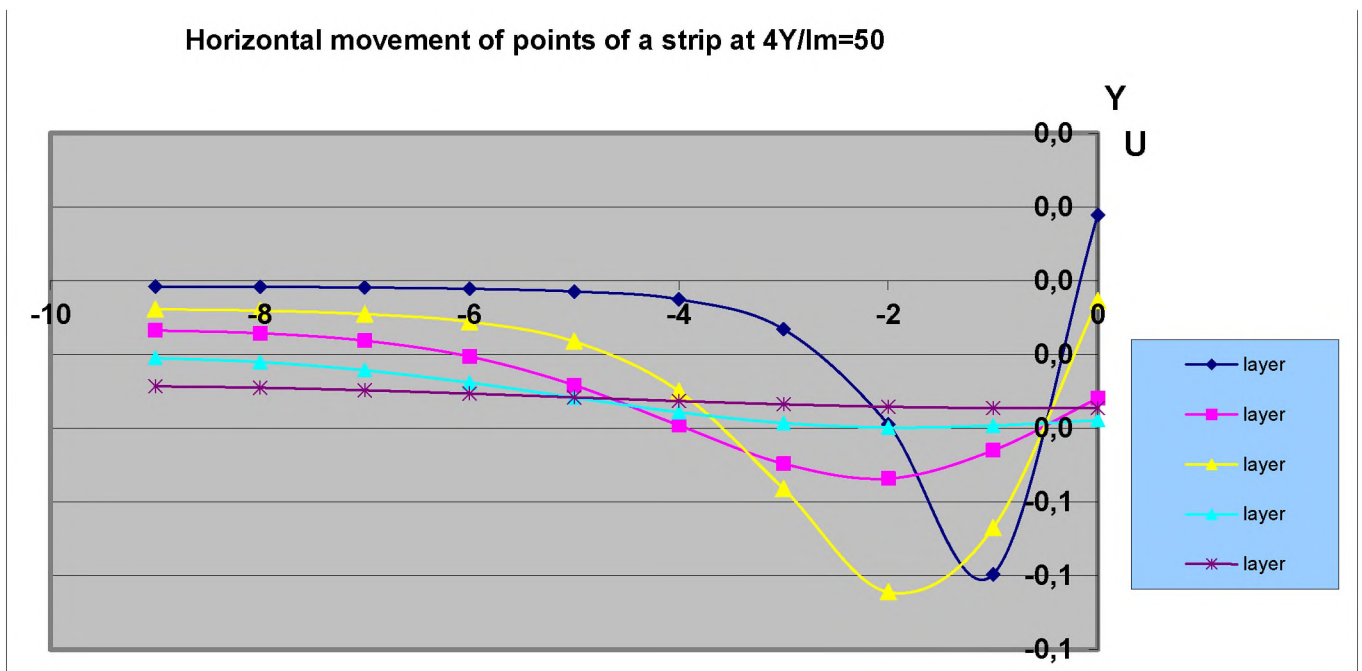The same phenomenon is observed for horizontal movements of the atoms located on vertical sections (Fig.10).

Horizontal movement of points of a strip at 4Y/lm=50

Fig.10. Charts of horizontal movements

The layer 1 (●——●) represents movement of atoms from intersection x= 0, and the layer 2 is ( ) is movement of atoms from intersection x = -λ, the layer 3is ( ) is movement of atoms in under x =-2λ and layer 4 - ( ) is movement of atoms in section x=-7 ρ.

In Fig.10 it is visible that the greatest horizontal movements are experienced by the atoms located in section x =0 at distance y =- (1÷2) ρ.

At different values horizontal movements also change identically, with variation in their values.

On the known components, deformation is easily calculated $\varepsilon_x = 1/\lambda * [U(x+ \rho,y)- U(x- \rho,y)]$ and $\varepsilon_y = 1/\lambda * [V(y+ \rho,x)- V(y- \rho,x)]$

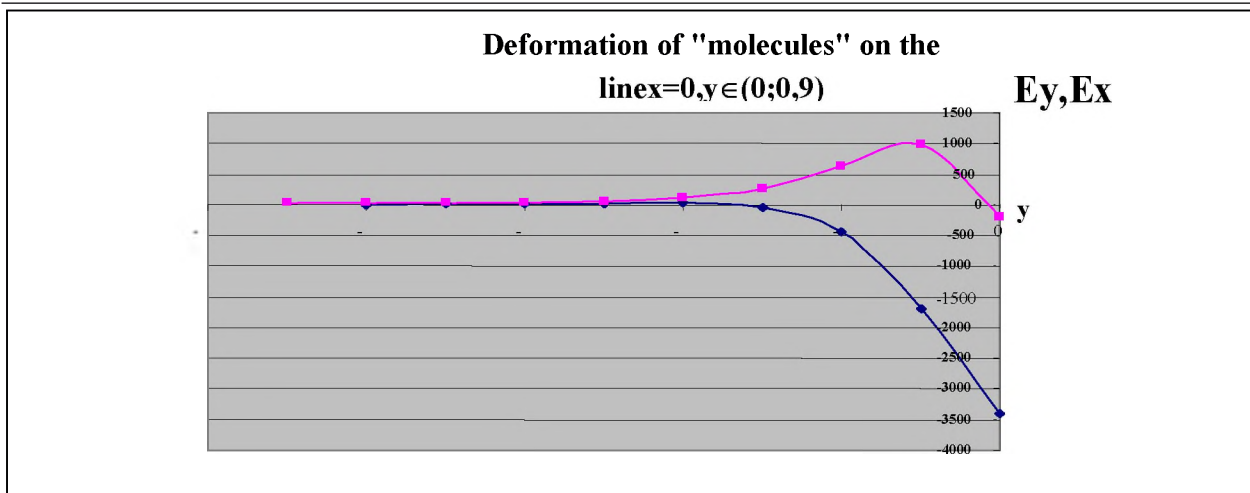The nature of their change in the intersection strip is shown on Fig.11 and 12.

Fig. 11. The chart of distribution of deformations $\varepsilon_x \varepsilon_y$ along the vertical axis of symmetry of a beam
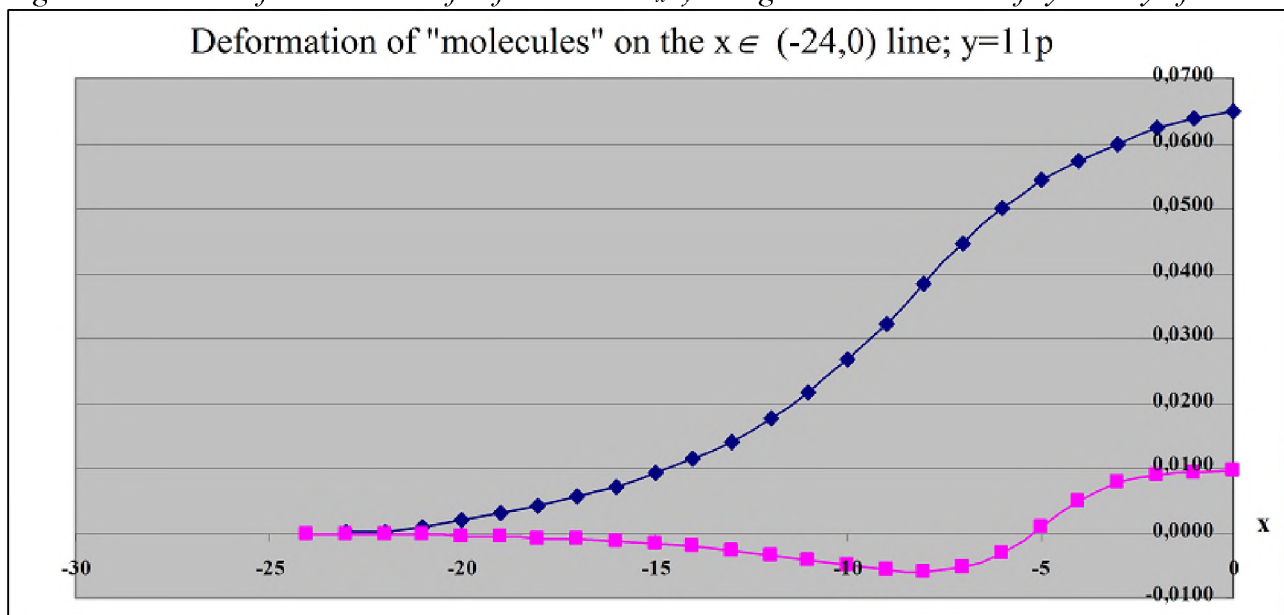

Fig. 12. The chart of distribution of deformations $\varepsilon_x \varepsilon_y$ along the horizontal axis of symmetry of a beam

At each value strength the greatest stretching deformation is tested by the "molecule" located under a point of application of force V.

The nature of change and the applied force is presented in fig. 13 and tab. 1.

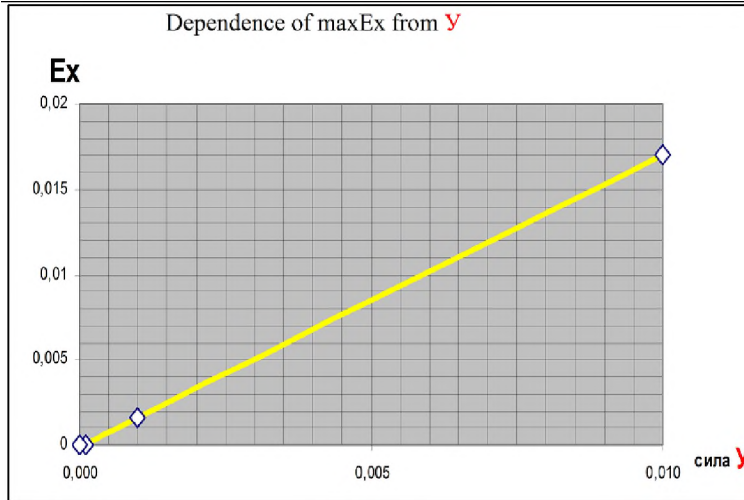| F	ε_x
1	1,582
0,5	0,791
0, 1	0,158
0,05	0,079
0,01	0,015

Fig. 13. The chart of change of deformation max ε_x from the applied force

From the chart it is possible to find that, value $\varepsilon_x = 0,01$ in particular is reached with the force $y=0.015$. Apparently this chart is linear. For linear approximation of this dependence of means of Excel - after recalculation, taking into account numerical these parameters of model and values strength y - is given dependence

$\varepsilon_x = 63,4 \ y/\mu\lambda \ (-0,003)$, in which free part (...) is necessary to consider as errors (iterative) in calculating movements/deformations and their approximation at small values of forces y.

Literature

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