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О  $\beta$ -ПОДОБНОЙ КОМПАКТИФИКАЦИИ РАВНОМЕРНЫХ ПРОСТРАНСТВ

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Получена характеристика базы  $\text{coz}$ -тонких равномерных пространств и для  $\beta$ -подобной компактификации доказан равномерный аналог теоремы Э. Чеха.

*Ключевые слова:*  $u$ -открытые,  $u$ -замкнутые множества;  $\text{coz}$ -отображение; нормальная база;  $\text{coz}$ -тонкое пространство.

ON  $\beta$ -LIKE COMPACTIFICATION OF THE UNIFORM SPACES

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The characterization of bases of  $\text{coz}$ -fine uniform spaces have been obtained and for  $\beta$ -like compactification the uniform analogue of E. Chekh theorem have been proved.

*Keywords:*  $u$ -open,  $u$ -closed sets;  $\text{coz}$ -mapping; normal base;  $\text{coz}$ -fine space.

**1. Introduction.** Denotations and basic properties of a uniform spaces and compactifications from [19], [2], [23]. We denote by  $U(uX)$  ( $U^*(uX)$ ) the set of all (bounded) uniformly continuous functions on the uniform space  $uX$ . The natural uniformity on  $uX$ , generated by  $U(uX)$  ( $U^*(uX)$ ), be  $u_c(u_p)$  is the smallest uniformity on  $X$  with respect to its all functions from  $U(uX)$  ( $U^*(uX)$ ) are uniformly continuous. Evidently,  $u_p \subseteq u_c \subseteq u_\omega \subseteq u$ , where a base of uniformity  $u_\omega$  is formed by all countable coverings of  $u$ . *Samuel compactification*  $s_u X$  is a completion of  $X$  with respect to uniformity  $u_p$ .  $Z_u$  is a ring of zero-sets of functions from  $U(uX)$  or  $U^*(uX)$  and  $CZ_u$  is a ring of cozero-sets of functions from  $U(uX)$  or  $U^*(uX)$ .  $CZ_u$  consists of complements of sets of  $Z_u$  and, vice versa. We note, that all sets of  $CZ_u(Z_u)$  coincide with set of all  $u$ -open ( $u$ -closed) sets in sense M. Charalambous [3], [4] of the uniform space  $uX$ .  $Z_u$  forms *separating, nest-generated intersection ring* on  $X$  and hence it is a *normal base* [9], [21], [13].

We denote the set of all natural numbers by  $\mathbb{N}$ ,  $\mathbb{R}$  is the real line, uniformity  $u_{\mathbb{R}}$  on  $\mathbb{R}$ , is induced by the ordinary metric; for  $X \subset Y$   $[X]_Y$  be a closure  $X$  in  $Y$ , for a compactum  $X$  we always use its a unique uniformity.

For *fine* uniformity  $u_f$  on Tychonoff space  $X$  [8], [19] every continuous function is uniformly continuous, hence  $U(u_f X) = C(X)$  ( $U^*(u_f X) = C^*(X)$ ) is the set of all (bounded) continuous functions on  $X$  and  $Z_{u_f} = Z(X)$  is the set of all zero-sets,  $CZ_{u_f} = CZ(X)$  is the set of all cozero-sets on  $X$  [8], [14]. Every maximal  $z_u$ -filter on  $Z_u$  is denoted as  $z_u$ -ultrafilter and  $z_{u_f}$ -ultrafilter on  $Z(X)$  is denoted as  $z$ -ultrafilter [14].

A covering of  $z$ -open sets is called  $u$ -open and a covering of cozero-sets is called *cozero-set covering*.

**Definition 1.1.** A mapping  $f: uX \rightarrow vY$  is called *coz-mapping*, if  $f^{-1}(CZ_v) \subseteq CZ_u$  (or  $f^{-1}(Z_v) \subseteq Z_u$ ) [10], [11]. A mapping  $f: uX \rightarrow Y$  of the uniform space  $uX$  into Tychonoff space  $Y$  is called  $z_u$ -continuous, if  $f^{-1}(CZ(Y)) \subseteq CZ_u$  (or  $f^{-1}(Z(Y)) \subseteq Z_u$ ) [7].

Evidently, that every uniformly continuous mapping is a *coz-mapping* and converse, generally speaking, incorrectly [4], [5]. Also, every  $z_u$ -continuous mapping  $f: uX \rightarrow Y$  is *coz-mapping* of  $f: uX \rightarrow vY$  for any uniformity  $v$  on  $Y$ . If  $Y$  is a Lindelöf or  $(Y, \rho)$  is a metric space, then its *coz-mapping* is a  $z_u$ -continuous (see. for example, [4], [5]). If  $Y = \mathbb{R}$  is a real number set, or  $Y = I = [0, 1]$  is a unit interval *coz-mapping* of  $f: uX \rightarrow \mathbb{R}$  is called  *$u$ -continuous function* and *coz-mapping* of  $f: uX \rightarrow I$  is called  *$u$ -function* [4], [5].

We denote as  $C_u(X)$  ( $C_u^*(X)$ ) the set of all (bounded)  $u$ -continuous functions on the uniform space  $uX$  and  $Z(uX)$  be a ring of zero-sets functions from  $C_u(X)$  or ( $C_u^*(X)$ ) and  $CZ(uX)$  consists of complements of sets of  $Z(uX)$  and, vice versa.

We formulate some statements without proofs, proved by A. A. Chekeev in his paper “Uniformities for Wallman compactifications and realcompactifications”, it is submitted and will be published in journal “Topology and its Applications”.

**Proposition 1.2.** *On uniform spaces  $uX$  the set  $\mathcal{B}_p^*$  ( $\mathcal{B}_\omega^*$ ) of all finite (countable)  $u$  – open coverings is the base of uniformity  $u_p^z$  ( $u_\omega^z$ ). Moreover  $u_p \subseteq u_p^z$ ,  $u_p \subseteq u_c \subseteq u_\omega \subseteq u_\omega^z$ .*

**Proposition 1.3.**  *$C_u(X)$  forms a complete subring of  $C_u(X)$  with inversion. It contains constant functions, separates points and closed sets, is uniformly closed and is closed under inversion, i.e. if  $f \in C_u(X)$  and  $f(x) \neq 0$  for all  $x \in X$  then  $1/f \in C_u(X)$  (and an algebra in sense of [15], [16], [18]).*

**Lemma 1.4.**

- (1) *coz – mapping  $f: uX \rightarrow vY$  into a compact space  $vY$  is uniformly continuous mappings  $f: u_p^z X \rightarrow vY$ ;*
- (1') *coz – mapping  $f: uX \rightarrow vY$  into  $\mathfrak{S}_0$  – bounded uniform space  $vY$  is uniformly continuous mappings  $f: u_\omega^z X \rightarrow vY$ .*
- (2)  *$U(uX) = U(u_c X) = U(u_\omega X) \subset U(u_p^z X) = C_u(X)$ ;*
- (2')  *$U(u_p X) = U^*(uX) \subset U(u_c^z X) = U^*(u_\omega^z X) = C_u^*(X) \subset C_u(X)$ .*
- (3)  *$Z_u = Z_{u_p} = Z_{u_c} = Z_{u_\omega} = Z_{u_p^z} = Z_{u_\omega^z} = Z(uX)$ .*
- (4)  *$!_u(X)$  is a complete ring of functions with inversion on  $X$ .*

**Corollary 1.5.** (1)  *$u_p^z$  is the smallest uniformity on  $X$  with respect to which coz – mapping into a compactum  $vY$  is uniformly continuous.*

(2)  *$u_\omega^z$  is the smallest uniformity on  $X$  with respect to which every coz – mapping  $f: uX \rightarrow vY$  into an  $\mathfrak{S}_0$  – bounded uniform space  $vY$  is uniformly continuous.*

Let  $\omega(X, Z_u)$  be a Wallman compactification of with respect to the normal base  $Z_u$  [9]. We note that  $\omega(X, Z_u)$  is  $\beta$  – like compactification of  $X$  [20] and put  $\beta_u X = \omega(X, Z_u)$ .

**Theorem 1.6.** *For a uniform space  $uX$  the following compactifications of  $X$  coincide:*

- (1) *The completion of  $X$  with respect to  $u_p^z$ .*
- (2) *The Wallman compactification  $\omega(X, Z_u)$  of  $X$  with respect to the normal base  $Z_u$ .*
- (3) *The compactification which is the set of all maximal ideals of  $C_u^*(X)$  equipped with Stone topology [22].*

**Corollary 1.7.** *Every coz – mapping  $f: uX \rightarrow vY$  can be extended to the continuous mapping  $\beta f: \beta_u X \rightarrow \beta_v Y$ . The first axiom of countability doesn't hold in any point  $x \in \beta_u X \setminus X$ . For uniform spaces  $uX$  and  $u'X$  we have  $\beta_u X = \beta_{u'} X$  if and only if  $Z_u = Z_{u'}$ .*

**Theorem 1.8.** *For a uniform space  $uX$  the following conditions are equivalent:*

- (1) *Samuel compactification  $s_u X$  of  $uX$  is a  $\beta$  – like compactification of  $X$ ;*
- (2)  *$u_p = u_p^z$ ;*
- (3) *any coz – mapping  $f: uX \rightarrow K$  into a compactum  $K$  can be extended to  $s_u X$ ;*
- (4) *any  $u$  – function  $f: uX \rightarrow I$  into a  $I$  can be extended to  $s_u X$ ;*
- (5) *if  $Z_1, Z_2 \in Z_u$  and  $Z_1 \cap Z_2 = \emptyset$  then  $[Z_1]_{s_u X} \cap [Z_2]_{s_u X} = \emptyset$ ;*
- (6)  *$[Z_1]_{s_u X} \cap [Z_2]_{s_u X} = [Z_1 \cap Z_2]_{s_u X}$  holds for any  $Z_1, Z_2 \in Z_u$ ;*
- (7) *every point of  $s_u X$  is the limit point for a unique  $z_u$  – ultrafilter on  $uX$ ;*
- (8) *every  $z_u$  – ultrafilter is a Cauchy filter with respect to  $u_p$ .*

## 2. On compactifications of coz-fine uniform spaces.

**Definition 2.1.** [10] A uniform space  $uX$  is called *Alexandroff space* if its each finite  $u$  – open covering is uniform.

**Theorem 2.2.** *For a uniform space  $uX$   $s_u X = \beta_u X$  if and only if  $uX$  is an Alexandroff space.*

**Proof.** Let  $s_u X = \beta_u X$  for a uniform space  $uX$ . Then  $u_p = u_p^z$ , i.e. finite  $u$  – open covering is uniform, hence  $uX$  is an Alexandroff space.

Conversely, if a uniform space  $uX$  is an Alexandroff space then  $u_p = u_p^z$ , hence  $s_u X = \beta_u X$  (see Theorem 1.8). Q.E.D.

**Definition 2.3** [11] A mapping  $f: uX \rightarrow vY$  is called a *coz – homeomorphism*, if  $f$  maps  $X$  onto  $Y$  in a one-to-one way, and the inverse mapping  $f^{-1}: vY \rightarrow uX$  is *coz – mapping*. A two uniform spaces  $uX$  and  $vY$  are *coz – homeomorphic* to each other if there exists a *coz – homeomorphism* of  $uX$  onto  $vY$ .

The next theorem is a uniform analogue of E. Čech Theorem [6].

**Theorem 2.4.** *Let  $uX$  and  $vY$  be the first-countable uniform spaces. Then  $uX$  is coz – homeomorphic to  $vY$  if and only if  $\beta_u X$  is homeomorphic to  $\beta_v Y$ .*

**Proof.** If  $uX$  is coz – homeomorphic to  $vY$ , then evidently, that  $\beta_u X$  is homeomorphic to  $\beta_v Y$  (Corollary 1.7).

Conversely, if  $\beta_u X$  is homeomorphic to  $\beta_v Y$ , then the uniform spaces  $u_p^z X$  and  $v_p^z Y$  are uniformly homeomorphic to each other (items 1, 2 of Theorem 1.8) and all points with the first-countability axiom of  $\beta_u X$  transferring to all points with the first-countability axiom of  $\beta_v Y$ , i.e.  $X$  is coz – homeomorphic to  $Y$  (Corollary 1.7, [1, Ch. IV, Ex.34]). Q.E.D.

**Definition 2.5** [10], [11]. A uniform space  $uX$  is called *coz – fine*, if each *coz – mapping*  $f: uX \rightarrow vY$  is uniformly continuous.

**Theorem 2.6.** [10], [11]. *For a uniform space  $uX$  the following conditions are equivalent:*

- (1)  $uX$  is a *coz – fine* space;
- (2)  $uX$  is a  $\mathcal{M}$  – fine and proximally fine space;
- (3) for every mapping  $f$  of  $uX$  into a uniform space  $vY$ , if  $f: uX \rightarrow v_p Y$  is uniformly continuous, then  $f: uX \rightarrow v_f Y$  is uniformly continuous, where  $v_f$  is a fine uniformity on  $Y$ ;
- (4) for every mapping  $f$  of  $uX$  into metrizable space  $vY$ , if  $f: uX \rightarrow v_p Y$  is uniformly continuous, then  $f: uX \rightarrow v_f Y$  is uniformly continuous, where  $v_f$  is a fine uniformity on  $Y$ ;
- (5)  $uX$  is a proximally fine Alexandroff space.

**Remark 2.7.** Information about of  $\mathcal{M}$  – fine and proximally fine uniform spaces see, for example, [10], [11], [17].

**Theorem 2.8.** *Let  $uX$  and  $vY$  be the first-countable coz – fine uniform spaces. Then  $uX$  is uniformly homeomorphic to  $vY$  if and only if  $\beta_u X$  is homeomorphic to  $\beta_v Y$ .*

**Proof.** It follows from Theorem 2.4 and Definition 2.5.

**Remark 2.9.** Remind, that  $u$  – open covering  $\alpha$  of a uniform space  $uX$  is called *coz – additive*, if  $\cup \alpha' \in Cz_u$ , for each  $\alpha' \subset \alpha$  [10], [11].

**Theorem 2.10.** *For uniform space  $uX$  the following conditions are equivalent:*

- (1)  $uX$  is *coz – fine*;
- (2) a family  $\mathcal{B}_u^*$  of all  $\sigma$  – locally finite completely *coz – additive*  $u$  – open coverings is a base of uniformity  $u$ ;
- (3) all locally finite *coz – additive*  $u$  – open coverings form a base of uniformity  $u$ .

**Proof.** (1)  $\Rightarrow$  (2). Evidently, for any  $\alpha, \beta \in \mathcal{B}_u^*$  a covering  $\alpha \wedge \beta$  is  $\sigma$  – locally finite completely *coz – additive*  $u$  – open. Hence  $\alpha \wedge \beta \in \mathcal{B}_u^*$ .

We prove a useful lemma for the continue proof of Theorem 2.10.

**Lemma 2.11.** *Let  $\{U_s = f_s^{-1}((0,1)) : s \in S\}$  be a point-finite  $u$  – open family, where  $f_s: uX \rightarrow I_s$  be a  $u$  – function and  $I_s = I$  for all  $s \in S$ . Then it induces a function  $f = \Delta\{f_s : s \in S\}: uX \rightarrow \mathbb{F}(S)$  is *coz – mapping*, where  $\mathbb{F}(S)$  denotes the subset of  $I^S = \prod\{I_s : s \in S\}$  consisting of all points  $x = (x_s : s \in S)$ , that have only a finite number of non-zero coordinates  $x_s$ .*

**Proof.** Evidently, that  $\mathbb{F}(S)$  is separable metrizable space. Then for any open set  $U \subset \mathbb{F}(S)$   $f^{-1}(U) = f_s^{-1}(p_s(U))$ , where  $p_s: \mathbb{F}(S) \rightarrow I_s$  is obvious projection. Then  $f^{-1}(U) \in Cz_u$ . Q.E.D.

Let  $\alpha = \{W_s : s \in S\}$  be an arbitrary  $\sigma$  – locally finite  $u$  – open covering. For each  $i \in \mathbb{N}$  a family  $\alpha_i = \{W_s : s \in S_i\}$  is locally finite  $u$  – open system. Then by Lemma 2.11 there exists *coz – mapping*  $f_i: uX \rightarrow \mathbb{F}(S_i)$ , where  $\mathbb{F}(S_i)$  is equipped with the metric  $d_i$  defined by  $d_i(x, y) = \sup\{|x_s - y_s|, s \in S_i\}$ . Each  $f_i$  is *coz – mapping* and so it is  $f = \Delta\{f_i : i \in \mathbb{N}\}: uX \rightarrow \prod\{\mathbb{F}(S_i) : i \in \mathbb{N}\}$ . Hence by item (1)  $f$  is uniformly continuous with respect to the metric uniformity  $v$  on  $\prod\{\mathbb{F}(S_i) : i \in \mathbb{N}\}$  and it is precompact reflection  $v_p$ . By item (4) of Theorem 2.6.  $f$  is uniformly continuous with respect to the fine uniformity  $v_f$  on  $\prod\{\mathbb{F}(S_i) : i \in \mathbb{N}\}$ , which is a metrizable space. Therefore fine uniformity  $v_f$  has a base consisting of all open coverings [19]. For each  $s \in S_i$  a set of the form  $f_s^{-1}(W)$ , where  $W$  is open in  $I_s$ , is the inverse image under  $f$ , is the open set  $p_i^{-1}(p_s^{-1}(W))$  of  $\prod\{\mathbb{F}(S_i) : i \in \mathbb{N}\}$ ,

where  $p_i : \prod \{\mathbb{F}(S_i) : i \in \mathbb{N}\} \rightarrow \mathbb{F}(S_i)$  and  $p_s : \mathbb{F}(S_i) \rightarrow I_s$  are the natural projections. A covering  $\beta = \{p_i^{-1}(p_s^{-1}(W)) : s \in S, i \in \mathbb{N}\}$  is uniform with respect to the fine uniformity  $\nu_f$ . Let  $\gamma$  be open  $\sigma$  – locally finite star-refinement of  $\beta$ . Then  $\gamma \in \nu_f$  and  $f^{-1}(\gamma)$  is the open  $\sigma$  – locally finite covering which is star-refined to  $\alpha$ .

(2)  $\Rightarrow$  (3). It is obvious.

(3)  $\Rightarrow$  (1). Let  $f : uX \rightarrow vY$  is uniformly continuous mapping into a metrizable uniform space  $vY$ . Then the mapping  $f : uX \rightarrow v_p Y$  is also uniformly continuous. By item (3) of Lemma 1.3. and metrizability of  $vY$  we have  $Z_v = Z_{v_p} = Z(Y)$ . The fine uniformity  $\nu_f$  of the metrizable uniform space  $vY$  has a base consisting of all open locally finite coverings, hence,  $f : uX \rightarrow v_p Y$  is also uniformly continuous mapping. Theorem is proved completely.

**Corollary 2.12** For a uniform space  $uX$  there exists such  $\text{coz}$  – fine uniformity  $u_{cf}$ , that  $u \subset u_{cf}$  and  $u_\omega^z \subset u_{cf}$ .

**Proof.** A uniformity  $u$  has a base of some family of  $\sigma$  – uniformly discrete completely  $\text{coz}$  – additive  $u$  – open coverings, hence  $u \subset u_{cf}$ . Every countable  $u$  – open covering is a  $\sigma$  – locally finite  $u$  – open covering, hence  $u_\omega^z \subset u_{cf}$ . Q.E.D.

**Corollary 2.13** Every Cauchy  $z_u$  – ultrafilter with respect to uniformity  $u_{cf}$  is countably centered.

**Proof.** It follows from  $u_\omega^z \subset u_{cf}$ .

**Corollary 2.14.** The completion  $\mu_u X$  of the uniform space  $uX$  with respect to the uniformity  $u_{cf}$  is contained in the Wallman realcompactification  $v_u X$ , i.e.  $\mu_u X \subset v_u X \subset \beta_u X$ .

**Proof.** It follows immediately from Corollary 2.13.

**Corollary 2.15.** Let  $w$  be a uniformity of completion  $\mu_u X$ . Then  $\beta_w(\mu_u X) = \beta_u X$ .

**Proof.** It follows immediately from Corollary 2.14.

**Theorem 2.16.** Let  $uX$  and  $vY$  be first-countable  $\text{coz}$  – fine uniform spaces. Then  $\alpha_u X$  is uniformly homeomorphic to  $\mu_v Y$  if and only if  $\beta_u X$  is homeomorphic to  $\beta_v Y$ .

**Proof.** Let  $w$  and  $w'$  be a uniformity of completions  $\mu_u X$  and  $\mu_v Y$ , respectively. Then  $\beta_w(\mu_u X) = \beta_u X$  and  $\beta_{w'}(\mu_v Y) = \beta_v Y$  and  $\beta_u X$  is homeomorphic to  $\beta_v Y$ , if  $\mu_u X$  is uniformly homeomorphic to  $\mu_v Y$ .

Conversely, from homeomorphy of  $\beta_u X$  and  $\beta_v Y$  it follows uniform homeomorphy of  $uX$  and  $vY$  (see Theorem 2.8). Then completions  $\mu_u X$  and  $\mu_v Y$  are uniformly homeomorphic to each other. Q.E.D.

**Corollary 2.17** Let  $uX$  and  $vY$  be complete the first-countable  $\text{coz}$  – fine uniform spaces. Then  $uX$  is uniformly homeomorphic to  $vY$  if and only if  $\beta_u X$  is homeomorphic to  $\beta_v Y$ .

**Proof.** It follows immediately from Theorem 2.16.

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