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RESISTANCE OF BUILDINGS TO SEISMIC IMPACTS BASED ON THE THEORY OF LINEAR VIBRATIONS

РЕЗИСТЕНТНОСТЬ ЗДАНИЙ К СЕЙСМИЧЕСКИМ ВОЗДЕЙСТВИЯМ НА ОСНОВЕ ТЕОРИИ ЛИНЕЙНЫХ КОЛЕБАНИЙ

Макалада сейсмикалык термелүүлөрдүн теориялык негиздери жана имараттардын жана курулмалардын туруктуулугуна, ошондой эле курулуш конструкцияларынын ташуучу элементтерине алардын таасири каралат. Жер титирөөлөрдө козголуучу сызыктуу термелүүлөр мүнөздөлгөн, курулуш конструкцияларынын эсептөө схемалары жана динамикалык процесстердин да, конструкцияларда өнүккөн инерциялык реакциялардын да сандык үлгүлөрү келтирилген. Сейсмикалык күчтөрдүн таасири алдында имараттардын жана курулмалардын резистенттүүлүгүн арттырууну мүмкүн кылган тыянактар алынган.

Өзөк сөздөр: *курулуш конструкциясы, конструкциянын ташуучу элементи, сейсмикалык таасир, сызыктуу термелүү, инерциялык реакция, деформация, көтөрүү жөндөмдүүлүгү, серпилгичтик, катуулук.*

В статье рассматриваются теоретические основы сейсмических колебаний и их воздействие на устойчивость зданий и сооружений, а также на несущие элементы строительных конструкций. Описаны линейные колебания, возбуждаемые при землетрясениях, приведены расчетные схемы строительных конструкций и численные модели как динамических процессов, так и инерционных реакций, развивающихся в теле конструкций. Получены выводы, которые позволят повысить резистентность зданий и сооружений при действии сейсмических сил.

Ключевые слова: *строительная конструкция, несущий элемент конструкции, сейсмическое воздействие, линейное колебание, инерционная реакция, деформация, несущая способность, упругость, жесткость.*

The article discusses the theoretical foundations of seismic vibrations and their impact on the stability of buildings and structures, as well as on the bearing elements of building structures. Linear oscillations excited during earthquakes are described, design schemes of building structures and numerical models of both dynamic processes and inertial reactions developing in the body of structures are provided. Conclusions have been obtained that will improve the resistance of buildings and structures under the action of seismic forces.

Keywords: *building structure, structural element, seismic action, linear oscillation, inertial reaction, deformation, bearing capacity, elasticity, rigidity.*

Relevance. Seismic forces are a complex of dynamic structures of a spatial nature. Hig loads on buildings and structures cause seismic deformations that go beyond the elastic and stiffness parameters of the structures themselves. They are non-linear type. Seismic forces based on the theory of linear development can be observed through three mutually opposite wave components. For the purpose of mathematical analysis of the seismic load component, the load-bearing structures can be replaced by an anti-gravity numerical rod model with the corresponding loads attached to it. The authors recommend numerical models of structures with a limiting number of loads, or with a linear development.

Research Purpose. Make a theoretical justification for the practice of using buildings and structures resistant to seismic forces.

Methodology. The following methods of structural mechanics were used in the work: the method of concentrated deformations, the method of boundary conditions, methods of graphic statics, methods of statically determinate and statically indeterminate systems, fundamentals of the seismic vibrations theory.

Research Results. The theory of seismic resistance of buildings and structures provides calculation schemes according to the type of discrete and continuous topologies, which are shown in Fig. 1 and 2. In particular, the layout configuration of a rectangular cross-section having a variable stem can be assumed to be flexible and change in various variations in shapes [1, 2, 3].

Seismic vibrations in various calculations used to determine the inertial force are described above. We will consider how these equations are constructed regarding the structure of the foundation of a building system.

Changes in time $Y=Y(t)$ is considered as given. An oscillatory system with this structure as a seismic effect on the behavior and development of its properties and irreversible energy absorption capacity is completely determined by inertial parameters.

The discrete model is determined (see Fig. 1) by the pinching of loads mk and the characteristics of the inertia of the points of their pinching ($k = 1,2,3, \dots n$).

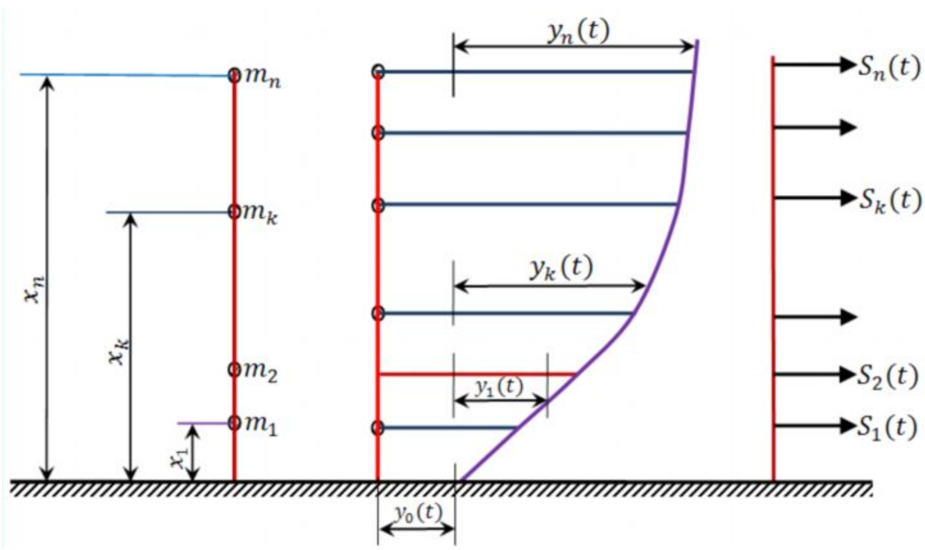

Fig. 1. Calculation scheme of a discrete type

Deformations develop through boundary displacements δkv, which have concentrated loads at the points of rigid pinching, visualized by a square symmetric matrix of the form $\delta=[\delta kv]^n$.

In each time range, the limiting state of the discrete numerical model is determined by displacements $yk\ (t)$ relative to concentrated loads from a quiet state. The quantitative indication of these coordinates is represented by the number of degrees of freedom of the Yk system, whose mathematical functions are the determining factors of the problem. The number n of these

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coordinates is the number of degrees of freedom of the system. The functions *Yk* are necessary factors of the problem [1].

$$S_k(t) = -m_k \left[y_k(t) + Y_0(t) \right] \tag{1}$$

$(k=1,2,3,\dots,n).$

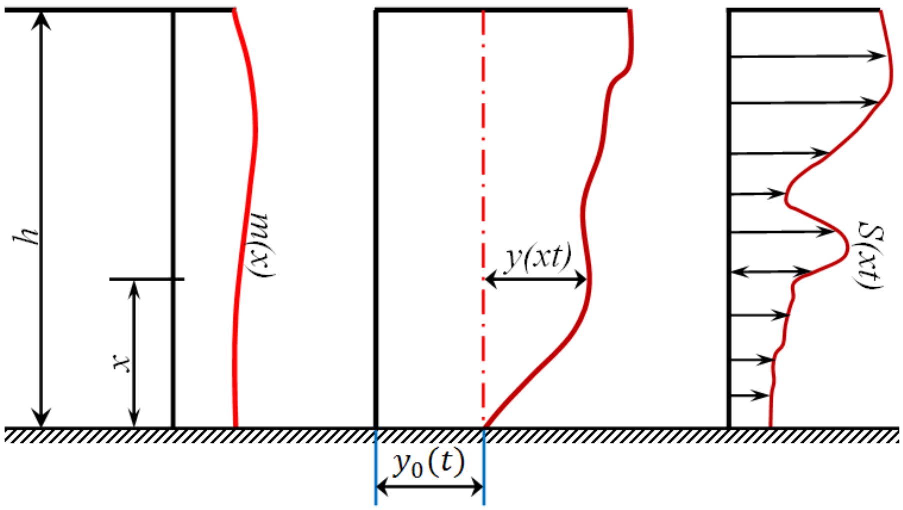

Fig. 2. Numerical model with distributed parameters

The indices in square brackets on the right of formula (1) show the acceleration of the compression point of the solid mass relative to the inertia coordinate system.

We write the differential equations of the wave system as a displacement of points under the action of inertia forces. Linearity of the system and single changes due to the use of δkv [1]

$$y_k(t) = -\sum_{v=1}^{n} m_v \left[y_v(t) + Y_0(t) \right] \delta_{kv} \tag{1}$$

wherefrom we get the following

$$\sum_{v=1}^{n} m_v \delta_{kv} y_v(t) + y_k(t) = -Y_0(t) \sum_{v=1}^{n} m_v \delta_{kv} \tag{2}$$

$(k=1,23,\dots,n).$

Formula (2) shows the differential equations of seismic forces and the calculation system of the discrete model.

The initiation of seismic excitation is written as an equilibrium phase at $t = 0$ or the absence of displacement and velocity of the system at the moment of excitation of the seismic force:

$$y_k(0) = 0; (k = 1,2,3,\dots n)\,. \tag{3}$$

The mathematical expression (2), taking into account (3), determines the functions *yk (t)* from the seismic resistance of the structures. According to the theory of linear and non-linear differential equations, the solution can be shown as follows:

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$$y_k(t) = -\sum_{i=1}^{n} \frac{D_i}{2\pi} T_i X_{ik} \int_0^1 Y_0(\tau) \sin \frac{2\pi}{T_i}(t-\tau) d\tau \tag{4}$$

(k=1,2, …,n),

where *Ti* is the temporal characteristic of the natural oscillations of the system;
Xik is amplitude coefficients that characterize the forms of natural oscillations;
Di is expansion coefficients given by expressions

$$D_i = \frac{\sum_{k=1}^{n} m_k X_{ik}}{\sum_{k=1}^{n} m_k X_{ik}^2} (i = 1,2 \ldots, n). \tag{5}$$

Differential indexer *L*, which determines the volumetric weight of the distributed load, which compensates for deformation phenomena:

$$L[y(x)] = q(x). \tag{6}$$

In the process of excitation of seismic energy, distributed loads are applied to the supporting structure with a force

$$s(x,t) = -m(x)\lfloor y(x,t) + Y_0(t) \rfloor. \tag{7}$$

This force at each moment of time compensates for the deformation *y(x, t)*. Based on (6)

$$L[y(x,t)] = -m(x)\lfloor y(x,t) + Y_0(t) \rfloor$$

Herefrom we obtain the differential equation of seismic vibrations of the continuum design scheme without taking energy dissipation into account:

$$m(x) y(x,t) + L[y(x,t)] = -m(x) Y_0(t). \tag{8}$$

The initial conditions can be written as follows:

$$y(x,0) \equiv 0, y(x,0) \equiv 0. \tag{9}$$

The solution of a linear inhomogeneous partial differential equation (8) can be found by the diffraction of variables (Fourier) method with the distribution of the right component over eigenfunctions in one series. Under initial conditions (9), we write this solution as follows:

$$y(x,t) = -\sum_{i=1}^{\infty} \frac{D_i}{2\pi} T_i X_i(x) \int_0^t Y_0(\tau) \sin \frac{2\pi}{T_i}(t-\tau) d\tau. \tag{10}$$

Here *Xi(x)* are eigenfunctions that determine the forms of natural oscillations of the Building structure. The distribution coefficients *Di* in this case are written by the following formula

$$D_i = \frac{\int_0^h m(x) X_i(x)\,dx}{\int_0^h m(x) X_i^2(x)\,dx}. \tag{11}$$

The above solutions do not take into account, as noted above, the dynamic effect in oscillatory systems, expressed by irreversible absorption or dissipation of the energy of the excited seismic vibrations.

In works on the theory of seismic resistance of building structures to seismic loads, energy dissipation, as a rule, is taken into account in accordance with the theory of viscous resistance (Focht Theory). In this regard, additional dissipating forces are introduced into the differential equations of seismic vibrations, which are proportional to the vibration velocities and directed in the opposite direction [1, 2].

In order to take into account the dissipation of seismic energy, according to Sorokin Theory, in the differential equations of seismic vibrations, all horizontal and vertical displacements and forces are written by means of a complex of mathematical functions and to the weights with the inclusion of an additional factor (*1 + Yi*), where *i* is an imaginary unit [4].

Taking into account the dissipation of seismic energy in both of the above methods in the problems of seismic vibrations leads to the fact that additional exponential parameters appear in the expressions of the integral formulas for solutions (4), (10). In the future, the authors will follow E.S. Sorokin Theory [4], which is the most physically substantiated. According to Sorokin Theory [4], the solutions to the problems of seismic vibrations, supplemented with allowance for energy dissipation, can be written in the following form (for a discrete calculation scheme):

$$y_k(t) = -\sum_{i=1}^{n} \frac{D_i}{2\pi} T_i X_i \int_0^t Y_o(\tau) e^{-\frac{\gamma\pi}{T_i}(t-\tau)} \sin\frac{2\pi}{T_i}(t-\tau)\,d\tau \tag{12}$$

(k=1,2, ...,n).

For a scheme with distributed parameters

$$y(x,t) = -\sum_{i=1}^{\infty} \frac{D_i}{2\pi} T_i X_i(x) \int_0^t Y_0(\tau) e^{-\frac{\gamma\pi}{T_i}(t-\tau)} \sin\frac{2\pi}{T_i}(t-\tau)\,d\tau. \tag{13}$$

It should be noted that the inclusion of seismic energy dissipation according to Sorokin Theory does not affect the characteristics of building structures, therefore all these values, as well as the coefficients *Di* in expressions (12), (13) have the same value.

Expressions (12), (13) fully define the law of seismic vibrations of building structures for a given law of movement of foundation soils. As you can see, these expressions are structurally identical, point offsets are presented as a set of elements of the same type. In a numerical model with distributed parameters, the number of these indices is unlimited.

With a discrete scheme, we have a finite sum of terms, the number of which is identical to the number of degrees of freedom of the system. Each term in expressions (12), (13) describes the normal vibrational work, in which all points of Building structures are displaced synchronously, and the form of vibration does not change in time. For the *i*-th normal oscillation, this form is described by the eigenfunction *Xi(x)* in the case of a continuous circuit and by a set of amplitude coefficients *Xik (k = 1,2, ..., n)* in the case of a discrete design scheme. The latter show the location of the vibration form on the graph at individual mass contact points. The general pattern of changes in displacements at any time during the *i*-th oscillation is written by an expression called the Duhamel's integral:

$$I_i(t)=\int_o^t Y_o(\tau)e^{-\frac{\gamma\pi}{T_i}(t-\tau)}\sin\frac{2\pi}{T_i}(t-\tau)d\tau\,. \tag{14}$$

The normal components in the sum (12) or in the series (13) are determined by comparing the products *DiTi* and the integrals *Ii(t)*. Here the first few terms are decisive.

Thus, formulas (12), (13) represent displacements of points of building structures in the process of seismic vibrations in the form of a distribution into normal components. Other seismic load factors are presented in the same form [5, 6]. In a discrete design scheme, the forces concentrated at a point of inertia, applied at points *k*, are expressed by the following formula

$$S_k(t)=\sum_{i=1}^{n} S_{ik}(t)=-\sum_{i=1}^{n} m_k\frac{2\pi D_i X_{ik}}{T_i}I_i(t) \tag{15}$$

(k=1,2,…,n).

With a continual design scheme for the intensity of distributed forces along the line and/or area of inertia, we write the following:

$$s(x,t)=\sum_{i=1}^{\infty} S_i(x,t)=-\sum_{i=1}^{\infty} m(x)\frac{2\pi D_i X_i(x)}{T_i}I_i(t)\,. \tag{16}$$

Here *Si* are concentrated and distributed inertial loads for individual normal vibrations.

What is interesting, from the point of view of the analysis of seismic forces, is a particular case of a discrete numerical model – a linear oscillator, which has one concentrated load (Fig. 3).

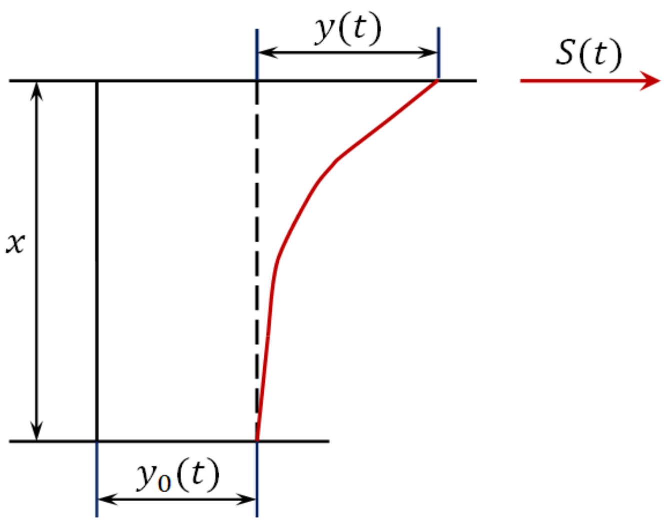

Fig. 3. Numerical model of the oscillator

Let's assign its weight and oscillation period through *m* and *T*. In this case, according to expression (5), we can take *D=1* and *X11=1*. Then, based on formula (15), the strength of the seismic generator is determined as follows:

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$$S(t) = -m\frac{2\pi}{T}I(t)\,. \tag{17}$$

If we take the seismic force as the force of inertia, then in this formula the factor with the mass m is represented as the acceleration of the oscillator with the period *T* and the inelastic drag coefficient *Y* with respect to a fixed coordinate system. Let's write the following expression:

$$W(t,T,\gamma) = \frac{2\pi}{T}\int_{o}^{t} Y_0(\tau)e^{-\frac{\gamma\pi}{T}(t-\tau)} \times \sin\frac{2\pi}{T}(t-\tau)d\tau\,. \tag{18}$$

In the noted solutions of the problems, the dynamic properties of the load-bearing structural elements are represented by the forms/periods of natural oscillations, and the coefficient of negative elastic resistance *Y*.

For discrete design schemes, a square matrix is used to determine the periods and forms of natural oscillations, calculated as the product of the above-mentioned matrix of unit displacements δ and the opposed matrix of loads with elements *mk*:

$$\left[m_k\delta_{kv}\right]_{k,v}^{1,n} = \begin{matrix} m_1\delta_{11} & m_2\delta_{12} & \dots & m_n\delta_{1n} \\ m_1\delta_{21} & m_2\delta_{22} & \dots & m_n\delta_{2n} \\ \dots & \dots & \dots & \dots \\ m_1\delta_{n1} & m_2\delta_{n2} & \dots & m_n\delta_{nn} \end{matrix} \tag{19}$$

The dynamic coefficients of the wave height *Xik*, showing the forms of natural oscillations, are the coordinates of the corresponding vectors of the matrix noted above, the angular values along the wavelength of these oscillations are determined by its own parameters λi:

$$\varphi_i^0 = \frac{1}{\sqrt{\lambda_i}} \qquad (i = 1,2,\dots,n) \tag{20}$$

The angular values along the wavelength, taking into account the energy dissipation, are written by the following formula

$$\varphi_i = \frac{\varphi_i^0}{\sqrt{1+\frac{\lambda^2}{4}}}\,.$$

For Building structures, *Y* values do not exceed 0.20. Therefore, taking into account the dissipation of energy practically does not change the frequency, and we can take $\varphi i = \varphi i0$, in accordance with this, for the oscillation periods we will have as follows

$$T_i = \frac{2\pi}{\varphi_i} = 2\pi\sqrt{\lambda_i}\,. \tag{21}$$

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For circuits with linearly distributed characteristics, the functions *Xi(x)*, which are eigenfunctions and describe the waveforms, are obtained by solving the following ordinary differential equation:

$$L[X(x)] - \varphi 0^2 \, m(x) X(x) = 0 . \qquad (22)$$

The eigenvalues along the wavelength *φi0* are calculated here from the characteristic equation of the boundary value problem noted above. Frequencies *φi0* and periods *Ti* are determined by the above formulas.

Forms of natural oscillations of building structures differ from each other in the number of half-waves. The simplest form with the minimum number of half-waves corresponds to the first normal component (*i* = *1*). As the number *i* increases, the number of half-waves increases. For the modes of natural oscillations, the conditions of the Ortho regime are also identical, which describe the characteristics of the sovereignty of normal oscillations [6, 7, 1]. For discrete and continuous design schemes, these conditions can take the following form:

$$\sum_{\substack{k=1 \\ (i \neq j)}}^{n} m_k X_{ik} X_{jk} = 0; \int_0^h m(x) X_i(x) X_j(x) dx = 0 . \qquad (23)$$

The determination of the coefficient of inelastic resistance *Y*, which haracterizes the dissipation of energy in the formulas noted above, is based on taking into account the forces of internal friction in the material and is due to the hysteresis dependence between forces and displacements. Such a dependence is characteristic of real bodies (not fully elastic) under synchronous and asynchronous loads. Then the coefficient *Y* is determined by the following expression

$$\gamma = \frac{1}{2\pi} \frac{\Delta \Pi}{\Pi} . \qquad (24)$$

The practice of applying the theoretical calculations noted above and the conclusions obtained can be implemented using the example of [8, 9].

Conclusions. As experimental studies have shown, the value of *Y* does not depend on the wavelength of oscillations; in the range of operating voltages, it also does not depend on the height of the waves of the latter. All previous mathematical expressions are derived from the prediction that *γ=const*. The numerical values of *γ*, determined by the internal friction in the material, have been established experimentally in laboratory conditions. However, in buildings and structures, the vibration energy is dissipated not only because of this factor, but also because of friction in the nodes of the supporting structures and the reverse transfer of some of the seismic energy to the foundation soil.

The dynamic coefficient *Y* as the only parameter of seismic energy dissipation in the calculation formulas should reflect all types of energy losses. Therefore, its mathematical values for real building structures can be obtained by field and full-scale tests of the latter.

List of cited references:

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1. Imanaliev T.B. (Bolotbek T.). Seismic Resistance of Artificial Structures. – Bishkek: Ilim, 2010. – p. 210.
2. Kartsivadze G.N. Seismic Resistance of Road Artificial Structures. – M.: Transport, 1974. – p. 263.
3. Kartsivadze G.N., Medvedev S.V., Napetvaridze Sh.G. Earthquake-resistant Construction Abroad. – M.: Gosstroyizdat, 1962. – p. 223.
4. Sorokin E.S. On the Theory of Internal Friction During Vibrations of Elastic Systems. – M.: Gosstroyizdat, 1960. – p. 129.
5. Aizenberg Ia.M. Structures with Disconnected Connections for Seismic Regions. – M.: Stroyizdat, 1976. – p. 232.
6. Zavriev K.S., Nazarov A.G., Aizenberg Ia.M. Fundamentals of the Theory of Seismic Stability of Buildings and Structures. – M.: Stroyizdat, 1970. – p. 222.
7. Zavriev K.S. Earthquake Building Guide. – Tbilisi: Metsniereba, 1967. – p. 10.
8. Bolotbek T., Nasyrynbekova K.U., Saparbekov A.S., Zholbolduev A.P. Methods for Calculating Seismic-resistant Soil Structures/ Bulletin of the Kyrgyz State University of Construction, Transport and Architecture named after N. Isanova. 2020. No. 1 (67). pp. 133-139.
9. Bolotbek T., Temirkanova Zh.T., Nurlan uulu A., Toichu kyzy Zh., Askarova. Application of the Results of the Method of Concentrated Deformations to the Results of Numerical Experiments Based on the Finite Element Method/ Bulletin of the Kyrgyz State University of Construction, Transport and Architecture named after N. Isanova. 2020. No. 1 (67). pp. 140-146.