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ABOUT ONE APPROACH TO BOUNDARY-ELEMENT MODELING OF NONSTATIONARY GEOMECHANICAL PROBLEMS

ОБ ПОДХОДЕ К ГРАНИЧНО-ЭЛЕМЕНТНОМУ МОДЕЛИРОВАНИЮ НЕСТАЦИОНАРНЫХ ГЕОМЕХАНИЧЕСКИХ ЗАДАЧ

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Description: In the article the model of the rock mechanics problems solutions with account of rock mass nonstationary properties on the basis of the boundary-element method is offered. The advantage of the Boundary Element Method (solution of the boundary value problem) is an exact satisfaction of the original differential equation inside design area.

Keywords: Boundary elements method (BEM), Hooke's law, stresses, condition parameters, method.

Описание: В статье предложена модель решения задач механики горных пород с учетом нестационарных свойств массива горных пород на основе метода граничных элементов. Преимуществом метода граничных элементов (решение краевой задачи) является

точное удовлетворение исходного дифференциального уравнения внутри проектной области.

Ключевые слова: метод граничных элементов (БЭМ), закон Гука,

The difficult rheological behavior of rocks can be studied experimentally and theoretically. Experimentally the rheological properties are defined by test of rocks or at constant loading (simple creeping), or at constant deformation. The greatest distribution was received by tests at constant loading that is connected with considerable simplicity of experiment in comparison with tests for a relaxation of stresses. However, with a view of pedigree files forecasting behavior possibility reception, it is more expedient to use theoretical approaches with numerical realization. The theoretical method of research consists in a dependence establishment between the stresses operating on rocks caused by deformations, and their changes in time.

Research of rocks nonstationary properties and knowledge of processes course in a mass laws are eventually the important scientific and practical problems and can be used at the analysis of the is stress-deformed condition, and also for the decision of geodynamic research problems of landslips, collapses and other processes [5], which is characteristic for territory of Kyrgyzstan.

In this connection we put a problem of reception of the time factor account relations on the basis of use of a numerical method and algorithm of their decision construction.

As a research method the boundary elements method (BEM), proved the advantage before other numerical approaches the possibilities of reduction of investigated dimension area and the automatic account of conditions on infinity, is chosen.

In works [1,2] it is shown, that classical theories of plasticity and creeping can not to describe many prominent features of deformable bodies behavior.

By Hart [3] not elastic deformation is represented in the form of the following elastic, not elastic and temperature components speeds sum:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{n}_{ij} + \dot{\varepsilon}^{T}_{ij}, \qquad (1)$$

where

$$\dot{\varepsilon}^{n}_{ij} = f_{ij}(\sigma_{ij}, q^{(k)}_{ij}, T), \qquad (2)$$

$$\dot{q}^{(k)}_{ij} = \varphi_{ij}(\sigma_{ij}, q^{(k)}_{ij}, T), \qquad (3)$$

$$\dot{\varepsilon}^{n}_{kk} = 0, \qquad (4)$$

σ_{ij} - components of stresses tensor; $q^{(k)}_{ij}$ - condition parameters; T - temperature; the point over size means operation of a capture derivative of this size on time.

Nonlinear deformation $\dot{\varepsilon}^{n}_{ij}$ on Hart consists of two time-dependent components:

$$\varepsilon^{n}_{ij} + \varepsilon^{a}_{ij} + \varepsilon^{p}_{ij}, \qquad (5)$$

where ε^{a}_{ij} - so-called an elastic deformation meaning saved up deformation which reflects size and a direction of previous deformation history and completely disappears at unloading; ε^{p}_{ij} - depending on a way irreversible permanent deformation.

The plan of the creeping boundary-value problem decision we shell accept in a following order. Initial values of not elastic deformations ε^{n}_{ij} is assumed equal to zero, and initial values of condition parameters are set; the initial values of deformations and stresses are defined from the decision of a usual thermo elastic problem for the same body (with the same boundary conditions) for which not elastic problem interesting us is considered.

The thermo elastic problem can be solved by boundary elements method (BEM) [4]. Then from (1) - (4) we shell find speeds of not elastic deformations $\dot{\varepsilon}^{n}_{ij}$ and condition parameters $\dot{q}^{n}_{ij}$ at zero value of time.

At the basic of research the decision of Somyliyan’s equation lays:

$$\dot{u}_i(\xi)=\int_{\tilde{A}}\left[\dot{p}_j(x)U_j^{(i)}(\bar{\xi},x)-\dot{u}_j(x)P_j^{(i)}(\bar{\xi},x)\right]d\tilde{A}(x)+\int_S\dot{F}_j(\bar{x})U_j^{(i)}(\bar{\xi},\bar{x})dS(\bar{x})+ \\ +\int_S\left[Q_{jk}^{(i)}(\bar{\xi},\bar{x})\dot{\varepsilon}_{jk}^{n}(\bar{x})+\alpha L_{jk}^{(i)}(\bar{\xi},\bar{x})\dot{T}(\bar{x})\delta_{jk}\right]dS(\bar{x}),\quad (i,j,k=1,2) \tag{6}$$

where $\bar{\xi},\bar{x}\in S,\ x\in\tilde{A}$. Kernels $U_j^{(i)},P_j^{(i)}$ are presented in [4], and $Q_{jk}^{(i)},L_{jk}^{(i)}$ have a following kind:

$$Q_{jk}^{(i)}=-\frac{1}{4\pi(1-\nu)r}\left[(1-2\nu)(r_{,k}\delta_{ij}+r_{,j}\delta_{ik})-r_{,i}\delta_{jk}+2r_{,i}r_{,j}r_{,k}\right], \tag{7}$$

$$L_{jk}^{(i)}=-\frac{1}{4\pi(1-\nu)r}\left[(1-2\nu)(r_{,k}\delta_{ij}+r_{,j}\delta_{ik})-(1-3\nu)r_{,i}\delta_{jk}+2r_{,i}r_{,j}r_{,k}\right], \tag{8}$$

r – the distance between a point of load application and a field point.

In Hart's theory the $\dot{\varepsilon}_{ij}^{n}$ is depend only from σ_{ij}, $q_{ij}^{(k)}$ and T, therefore last in (6) integral on surface S can be estimated directly during any moment of time t as soon as the stresses, parameters of a condition and temperature are known. Hence, in our approach (on Hart) already there is no need in iteration, together with in the fluidity conditions and unloading criteria, as in the classical theory of plasticity. In the computing relation the stated in given work boundary-element formulation of the problem appears rather effective.

Solving (6) taking into account Hooke's law relations between loading and stresses components, we shell receive the speed of stresses:

$$\dot{\sigma}_{ij}(\bar{\xi})=-\int_B\left[\dot{p}_k(x)K_j^{(ij)}(\bar{\xi},x)-\dot{u}_k(x)N_k^{(ij)}(\bar{\xi},x)\right]dB(x)+ \\ +\int_S\dot{F}_j(\bar{x})K_k^{(ij)}(\bar{\xi},\bar{x})dS(\bar{x})+\int_S[M_{ki}^{(ij)}(\bar{\xi},\bar{x})\dot{\varepsilon}_{kl}^{n}(\bar{x})+ \\ +\alpha R_{kl}^{(ij)}(\bar{\xi},\bar{x})\dot{T}(\bar{x})\delta_{kl}]dS(\bar{x})-2G\dot{\varepsilon}_{kl}^{n}(\bar{\xi})-2K\alpha\dot{T}(\bar{\xi})\delta_{ij},\ (i,j,k,l=1,2), \tag{9}$$

where

$$K_k^{(ij)}=-\frac{1}{4\pi(1-\nu)r}\left[(1-2\nu)(r_{,j}\delta_{ik}+r_{,i}\delta_{jk}-r_{,k}\delta_{ij})+2r_{,i}r_{,j}r_{,k}\right], \tag{10}$$

$$N_k^{(ij)}=-\overline{M}_{kl}^{(ij)}n_l, \tag{11}$$

$$M_{kl}^{(ij)}=-\overline{M}_{kl}^{(ij)}+\frac{\nu G}{\pi(1-\nu)r^2}\left[2r_{,i}r_{,j}-\delta_{ij}\right]\delta_{kl}, \tag{12}$$

$$R_{kl}^{(ij)}=\overline{M}_{kl}^{(ij)}+\frac{\nu G}{\pi(1-\nu)r^2}\left[-2r_{,i}r_{,j}+\delta_{ij}\right]\delta_{kl}, \tag{13}$$

$$\overline{M}_{kl}^{(ij)}=-\frac{G}{2\pi(1-\nu)r^2}\begin{bmatrix}2(1-2\nu)(r_{,i}r_{,j}\delta_{kl}+r_{,k}r_{,l}\delta_{ij})+2\nu(r_{,k}r_{,j}\delta_{li}+r_{,l}r_{,i}\delta_{jk}+ \\ +r_{,l}r_{,j}\delta_{ik}+r_{,i}r_{,k}\delta_{jl})+(1-2\nu)(\delta_{ik}\delta_{lj}+\delta_{jk}\delta_{li})- \\ -(1-4\nu)\delta_{ij}\delta_{kl}-8r_{,i}r_{,j}r_{,k}r_{,l}\end{bmatrix}. \tag{14}$$

Directing any internal point ξ to a surface point x, we shell receive boundary integral equation (BIE):

$$C_{ij}\dot{u}_j(\xi)=\int_B\left[\dot{p}_j(x)U_j^{(i)}(x,\xi)-\dot{u}_j(x)P_j^{(i)}(x,\xi)\right]dB(x)+\int_S\dot{F}_j(\bar{x})U_j^{(i)}(\bar{x},\bar{\xi})dS(\bar{x})+ \\ +\int_S[Q_{jk}^{(i)}(\bar{\xi},\bar{x})\dot{\varepsilon}_{kl}^{n}(\bar{x})+\alpha L_{jk}^{(i)}(\bar{\xi},\bar{x})\dot{T}(\bar{x})\delta_{jk}]dS(\bar{x}),\quad (i,j,k=1,2), \tag{15}$$

As is known, BIE (15) in overwhelming majority of cases is solved numerically. We shell short result a way of their decision used by us in a considered case. The area S breaks on m cells (flat elements), designated by the centers of gravity x_m; the boundary (contour) of considered area breaks on N boundary elements with the centers of gravity ξ_M or x_N. The functions, $\dot{F}_j$, $\dot{\varepsilon}^n_{ij}$ and $\dot{T}$ we shell as constants within each cell, $\dot{u}_i$ and $\dot{p}_i$ - constants on each boundary element. As a result of such digitization BIE (15) is reduced in system of the linear algebraic equations (SLAE) (we accept the boundary B as regular):

$$\frac{1}{2}\dot{u}_i(\xi_M) = \sum_N \dot{p}_j(x_N)\Delta U_j^{(i)}(\xi_M, x_N) - \sum_N \dot{u}_j(x_N)\Delta P_j^{(i)}(\xi_M, x_N) + \dot{I}_i(\xi_M), \tag{16}$$

where

$$\dot{I}_i(\xi_M) = \sum_m F_j(\overline{x}_m) \int_{\Delta S_m} U_j^{(i)}(\xi_M, \overline{x}) dS(\overline{x}) + \sum_m \left[\dot{\varepsilon}^i_{jk}(\overline{x}_m) + \alpha \dot{T}(\overline{x}_m)\delta_{jk}\right] \times$$
$$\times \int_{\Delta S_m} Q_{jk}^{(i)}(\xi_M, \overline{x}) dS(\overline{x}), \quad (i, j, k = 1,2;\ M = 1,2,\ldots,N), \tag{17}$$

$$\Delta U_j^{(i)}(\xi_M, x_N) = \int_{\Delta B_N} U_j^{(i)}(\xi_M, x) dB(x), \tag{18}$$

$$\Delta P_j^{(i)}(\xi_M, x_N) = \int_{\Delta B_N} P_j^{(i)}(\xi_M, x) dB(x). \tag{19}$$

Required central speeds of displacements and tractions it is found from the equation

$$\sum_N \left(\frac{1}{2}\delta_{ij}\delta_{MN} + \Delta P_j^{(i)}(\xi_M, x_N)\right)\dot{u}_j(x_N) = \sum_N \Delta U_j^{(i)}(\xi_M, x_N)\dot{p}_j(x_N) + \dot{I}_j(\xi_M); \tag{20}$$

Or, in the matrix form,

$$[A]\{\dot{u}\} = [B]\{\dot{p}\} + \{\dot{I}\}, \tag{21}$$

where $[A] = \left(\frac{1}{2}[E] + [\Delta P]\right)$, $[B] = [\Delta U]$, $\{\dot{I}\} = \dot{I}_j$, $[E]$ - an unit matrix. (22)

Matrixes [A] and [B] have dimension 2Nx2N. Speeds of internal displacements and stresses are defined from discrete analogues of the equations (6) and (9) accordingly. We shell notice, that integrals (17)-(19) at the case when $\xi_M = x_N$ will be singular and can be calculated analytically in the closed kind for rectilinear boundary elements and polygonal internal cells.

The problems initial conditions it is found, setting initial distributions of condition parameters q_i^0 (which, in general, are functions of coordinates x_i), and, accepting initial not elastic deformation equal to zero, i.e. believing $\varepsilon^n_{ij} = 0$. Then at the case when t = 0 only elastic and temperature deformations will take place in a body. Hence, initial displacements u_i^0, stresses σ_{ij}^0 and deformations ε_{ij}^0 are defined by the decision of a corresponding thermo elastic problem or analytically (if it is possible), or numerically (for example, on BEM).

The speeds of displacements and stresses in a body at t = 0 it is found on (9) and (15), and speeds of condition parameters - on (3). Displacements, stresses and condition parameters during the following moment of time Δt it is found, for example, on Euler's method ($\sigma_{ij}\big|_{\Delta t} = \sigma_{ij} + \sigma_{ij}\big|_{t=0}$ etc.) or on a method of integration by Runge -Koutt of 4-th order type. Stresses $\sigma_{ij}(\Delta t)$ received on these methods and condition parameters of $q_i(\Delta t)$ it is used now for definition of speeds at Δt, and process continued further till demanded final time.

So, knowing the stresses and condition parameters at the time moment t, on the equations (2), (3), (9) and (15) we find speed during the same moment of time t. Then these speeds are used for definition of the subsequent stresses and condition parameters at (t + Δt) on suitably picked up scheme of integration on time. As a result we will receive time history of required unknown persons interesting us in all body.

Conclusions

In summary we shell notice, that in the given work the direct of boundary elements method for the creeping analysis plane deformed rigid bodies for which description is offered the Hart's theory is stated. On the basis of such model now we develop computer programs for the decision of difficult nonlinear problems of time-dependent not elastic deformation (creeping, plasticity) rigid bodies with any geometry of boundary.

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