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# ИНТЕРАКТИВНЫЕ ОБОБЩЕННЫЕ ЗАДАЧИ <br> В ПРЕПОДАВАНИИ МАТЕМАТИКИ 

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На основе определений случайно формируемых наборов задач и принципов «формируемости», «уникальности» в тестировании знаний, данных автором ранее, предлагаются такие задачи типов «черный ящик» и «операционные», которые могут быть реализованы только на компьютере.

Ключевые слова: обобщенная задача; интерактивная задача; преподавание; математика; компьютер.

## INTERACTIVE EXTENDED TASKS

## IN TEACHING MATHEMATICS

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On the base of definitions of randomly generated sets of tasks and principles of "generativity", "uniqueness" in testing knowledge given by the author earlier, such tasks of "black box" type and "operational type" (which can be implemented at a computer only) are proposed.
Keywords: extended task; interactive task; teaching; mathematics; computer.

Introduction. To use capacities of computer for teaching and monitoring of knowledge more fully we offered (see [1]).

Definition 1. An extended task is an algorithm generating different logically correct and methodically proper tasks (of same level of difficulty) and corresponding right answers from initial data, (randomly) chosen from finite but sufficiently large sets (ranges).

Remark. Another term for such object is a parameterized question.

Definition 2. An individually generated task consists of two algorithms.

The first algorithm composes tasks due to Definition 1 using personal information about a contestant. The second algorithm generates corresponding right answers using the same information.

Definition 3. An algorithm permitting one (a teacher) to choose subsets of sets of initial data and generating different logically correct and methodically proper tasks by initial data, (randomly) chosen from these subsets is said to be an adjustable extended task.

In this paper, we consider such type of extended tasks in mathematics which can be implemented by a computer only.

Definition 4. A complex computer examination is an adjustable software offering various extended tasks including traditional types of tasks, interactive tasks,
tasks with involving objects outside, etc. to score both knowledge, skills and keenness of wit on all aspects of the subject.

1. Some notions related to effective computer testing. To ensure the fulfillment of common demands of tests (Validity, Objectivity and Reliability), to improve efficiency of testing, we have offered the following principles:
> Generativity: a complete text (content) of a task must not exist before testing, and must be generated randomly just before it.
> Uniqueness: all students taking the test must obtain different versions of tasks (of the same level of difficulty or of different levels, according to the teacher's will).
To fulfill UNESCO's "The four pillars of education for the XXI Century" united by us as "Learning to do together" we implemented necessary-team competitions (teammates can communicate by means of a network only).
2. Examples of interactive extended tasks of "black box" type. The general task is to detect a "hidden" object by its responds to quests chosen by the user.

For integer numbers $p<q$, define random choosing an integer X fulfilling the condition $p \leq X \leq q$, as " $X \in p \ldots q$ ".

The following two examples are well-known, we cite them for completeness.

Task 1. "There is a hidden natural number less than 1000000 . Input numbers. You will be given responds GIVE MORE, GIVE LESS, YES! If you obtain YES! in less than 21 quests you will obtain full score".

Algorithm: $N \in$ 1000...999000; M: =0; repeat \{input $K$; $M:=M+1$; if $K<N$ then output "GIVE MORE"; if $K>N$ then output "GIVE LESS", if $K=N$ then \{output ("YES!’", M); exit\} \}

Task 2. "There is a hidden continuous function $F(X), F(0)<0, F(1000)>0$. Repeating input $X$ es, you will be given values $F(X)$. Solve the equation $F(X)=0$ with the accuracy 0.001 ..."

Remark. The random function $F(X)$ must be increasing but non-linear.

The following tasks were proposed at olympiads on applied mathematics conducted by the Kyrgyz-Russian Slavic University:

Task 3. 1) Output the text: '"There is a hidden cubic polynomial $F(X)=A X 3+B X 2+C X+D$ with integer coefficients. Find the expression $A 3 B+C 4 D$.

The fewer queries you make, the higher right answer will be estimated».
2) Let $A \in\{-3,-2,2,3\}, B \in-9 \ldots-1 \cup 1 \ldots 9, C$ $\in-9 \ldots-1 \cup 1 \ldots 9, D \in-9 \ldots-1 \cup 1 \ldots 9, M:=0$.
3) Output the text: «Input non-zero integer value of X in the range $100 \ldots 100$ or 0 for output».
4) Input $X$. If $X \neq 0$ then let $M:=M+1$, calculate and output $F(X)$ and go to 3 ).
5) If $X=0$ then output the text «Input $A 3 B+C 4 D »$.
6) Input $Y$. If $Y=A 3 B+C 4 D$ then output the text: "Congratulation: you have solved the task in 'M' queries!" else output the text: "Unfortunately, you are wrong".

Task 4. "There was a mixture of two radioactive isotopes with (sufficiently) different half-lives. Repeating input $T>0$, you will be given the mass of the mixture at time $T$ ( $T=0$ for exit). Find the ratio of mass of long-living isotope to one of short-living isotope at the beginning with an accuracy of 0.01 ."
3. Examples of interactive extended tasks of "operational" type. Mathematical tasks on drawing are not included into examinations because they are difficult to be organized and to be scored. But capacities of computers and skills of students to use computer mouse make it easy.

Task 5. Given some connected triangles. There are little sectors in all angles.
A) "Click on all obtuse angles!"
B) "Click on all right angles!"
C) "Click on all acute angles!"
D) "Click on all isosceles triangles!"

Task 6. Given a (very scalene) triangle PQR.
A) "Draw (approximately) the median from the vertex P until the side QR!"
B) "Draw (approximately) the bisector from the vertex P until the side QR!"
C) "Draw (approximately) the height from the vertex P until the side QR!"
D) "Click (approximately) on the center of the inscribed circle!"
E) "Click (approximately) on the center of the circumscribed circle!"

Task 7. Given some straight lines and points marked with capital letters.
A) Click (approximately) where is the point symmetric to point P with respect to the straight line QR!"
B) Click (approximately) where is the point symmetric to point $P$ with respect to the point Q !"

Task 8. Given some vectors of same origin O and points marked with capital letters.
A) Click (approximately) where will the point $P$ be after shifting by the vector OQ!"
B) Click (approximately) where will is the end point of vector OQ be after multiplying it by two (by 0.4 ), (after rotation of it counterclockwise by $60^{\circ} \ldots$ )"
C) Click (approximately) where will is the end point of vector ( $\mathrm{OQ}+\mathrm{OT}$ )!"

Task 9. Given a graph of a smooth non-monotonic function.
A) "Click on the point of maximum!"
B) "Click on the point of minimum!"
C) "Click on all points of local maxima!"
D) "Click on all points of local minima!"
E) "Click on all critical points!"

An example of task on keenness of wit:
Task 10. A) Add B) Erase C) Shift ... as few grey squares as possible to obtain a symmetric pattern (actually, the table should be about 5 r


Conclusion. Using the offered methodic, some examinations are conducted since 1996. Since 2006 Applied Mathematics and Informatics chair jointly with Information and Computing Technologies chair of the Kyrgyz-Russian Slavic University conducts open competitions at applied mathematics both for KRSU students and for students of other universities.

We hope that further developing of corresponding software would widen the scope of tasks and make mathematical examinations more effective.

## References

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